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The Position of the Sun's Axis as determined from photographs of the Sun from 1874 *to* 1912, *measured at the Royal Observatory, Greenwich.* (Second Paper.) By F. W. Dyson, M.A., LL.D., F.R.S., and E. W. Maunder.

1. In the earlier paper under this title communicated to the Royal Astronomical Society in May 1912, and appearing in the *Monthly Notices*, vol. lxxii. pp. 564–582, corrections were deduced to the assumed position of the Sun's axis as adopted in all the reductions of the measures of solar photographs made at Greenwich from 1874 up to the present time. This position was that determined by Carrington, viz.:

$$\mathrm{I} = 7^\circ\ 15'\ \text{and}\ \mathrm{N} = 73^\circ\ 40'\ \text{for}\ 1850{\cdot}0,$$

where I denotes the inclination of the Sun's equator to the ecliptic, and N the longitude on the ecliptic of its ascending node. If these require corrections $\Delta\mathrm{I}$ and $\Delta\mathrm{N}$ respectively, then, if

$$y = \Delta\mathrm{I} \quad \text{and} \quad x = \Delta\mathrm{N} \sin \mathrm{I},$$

it was shown that each spot-group observed for a sufficient period of time would yield a pair of equations for the determination of x and y of the following form:—

$$\left.\begin{aligned} x\cos\theta_1 - y\sin\theta_1 + l_1 &= z \\ x\cos\theta_2 - y\sin\theta_2 + l_2 &= z \end{aligned}\right\}$$

where θ_1, θ_2 are the longitudes of the spot-group reckoned on the ecliptic from the ascending node of the Sun's equator when the

group is near the East and West limbs of the Sun respectively, l_1, l_2 are the corresponding observed latitudes, and z is the *true* latitude of the group. These by subtraction give

$$x(\cos\theta_1 - \cos\theta_2) - y(\sin\theta_1 - \sin\theta_2) = -(l_1 - l_2).$$

All groups observed satisfactorily for 8 or more consecutive days were employed, and the means of the observations of longitude and of latitude for 3, 4, or 5 consecutive days, according to the length of time that the particular spot-group was under observation, were taken to form θ and l. Thus the means of the observations for the first 3 days and for the last 3 days were taken with groups lasting for 8, 9, or 10 days; with 11-day groups the means were taken for 4 days; and with groups of longer duration for 5 days. In the present paper, for a reason to be noted later, means of 4 days were taken with the 12-day groups, as well as with those of 11 days.

2. The material discussed in the earlier paper can be considerably increased by treating in a similar manner the groups of spots that return to the visible hemisphere two or more times. A catalogue of these for the years 1874 to 1906 was compiled by Mrs. Walter Maunder and published as an Appendix to the Greenwich Photo-Heliographic Results for 1906; and this has been continued in the Photo-Heliographic Results for the subsequent years. Just as the series of equations employed in the earlier paper were formed from the determinations of latitude made near the East limb and West limb respectively, in the case of a spot transiting across the visible hemisphere, so a similar pair of equations can be formed in the case of a group passing across the invisible hemisphere, from the determinations of latitude made near the West limb before it passed out of sight, and near the East limb after it has returned within view. In the present paper, therefore, all cases of Recurrent groups, where the group was satisfactorily observed for three consecutive days before disappearance at the West limb, and again for three consecutive days after reappearance at the East limb, have been used to form equations for the determination of the error of position of the assumed axis, the mean of 3 days' observations being taken in each case. This corresponds to the position of the group on the second day from the limb, and allowing 15 or 16 days for the time during which the group is wholly or in part in the further hemisphere, the two positions involve an interval of 18 or 19 days, as compared with the intervals of from 5 to 9 days given by the groups observed for 8 or more days in the nearer hemisphere. This longer interval implies that the Recurrent groups will be more strongly affected by systematic errors due to steady proper motion on the part of the spot-groups. But errors of orientation, on the other hand, will affect the equations derived from the Recurrent groups only to the same general extent as those derived from groups observed in the course of a single apparition.

The following is an example of the treatment of a Recurrent group :—

TABLE XV.

No. of Group.	Date.	Longitude of Sun from Node.	Longitude of Spot from Central Meridian.		Latitude of Spot.	
	1911. d					
6944	Apr. 23·386	...	−71·4		−7·1	
	24·433	...	−57·2		−6·7	
	25·113	...	−48·3		−6·8	
	26·425	...	−30·8		−6·3	
	27·611	...	−15·2		−6·0	
	28·528	...	− 2·8		−5·8	
	29·396	...	+ 8·7		−5·8	
	30·539	...	+24·1		−5·8	
	May 1·306	...	+34·3		−5·7	
	2·525	147°·5	+50·8	+62°·4	−5·8	−5°·73
	3·307		+61·4		−5·7	
	4·306		+75·1		−5·7	
Returning as						
6950	May 20·086	164°·8	−73·9	−57°·6	−6·7	−6°·50
	21·490		−54·9		−6·5	
	22·305		−44·1		−6·3	
	23·338	...	−30·8		−7·5	

Therefore, for the mean of the days May 2 to May 4 the longitude of the spot from the node = 147°·5 + 62°·4, or = 209°·9, and for the days May 20 to May 22 = 164°·8 − 57°·6, or = 107°·2 ; a difference of 102°·7 for an interval of 18 days. This gives the equations

$$0{\cdot}98\{x \cos 209^\circ{\cdot}9 - y \sin 209^\circ{\cdot}9\} - 5^\circ{\cdot}73 = +z$$
$$0{\cdot}98\{x \cos 107^\circ{\cdot}2 - y \sin 107^\circ{\cdot}2\} - 6^\circ{\cdot}50 = +z$$

giving by subtraction

$$-0{\cdot}56x + 1{\cdot}43y = -0^\circ{\cdot}77.$$

In this way the Recurrent groups yield a series of equations for the determination of x and y similar to those derived from the treatment of groups observed continuously in the course of the same apparition, as in the earlier paper.

The coefficients of x and y are, on the average, about as large as in the equations of the first paper, but the longer interval increases the effect of the proper motion of the spots, which, in these equations as in those considered before, is treated as accidental error.

3. A more important source of information is supplied by the observations of latitude made when a group of sun-spots is near

the Central Meridian of the disc. For the method described in the earlier paper, *Monthly Notices*, vol. lxxii. pp. 572 to 578, proceeds on the assumption that the difference in the apparent latitude of a spot-group near the East and West limbs may be treated as an accidental error except so far as it arises from an error in the position of the Sun's axis. The proper motion of each group is treated as an accidental error, and contributes the largest part of these differences in the apparent latitude, the quantities on the right-hand side of the equations, as in Table VIII. on p. 576.

But if the proper motion of a group of spots is uniform during its motion across the disc, an equation obtained by comparing the apparent latitude of a group in the middle of its course across the visible hemisphere, with the mean of the apparent latitudes when it is nearer the limb, will be free from the error introduced by such motion. Thus in the example given in Table VII. (p. 575 of the earlier paper), and reproduced below in Table XVI., the

TABLE XVI.

Spot-Group 2496.

Date. 1892	Longitude of Central Meridian from Node.	Longitude of Spot from Central Meridian.		Latitude of Spot.	
	d	°		°	
Apr. 27·52		−81·2		−23·8	
28·43		−71·8		−23·9	
29·40	145°·4	−58·8	−58°·0	−24·3	−24°·16
30·41		−45·8		−24·4	
May 1·42		−32·4		−24·4	
2·23		−21·9		−24·3	
3·40		− 6·4		−24·7	
4·21	149°·8	+ 3·8	− 1°·5	−25·1	−24°·92
5·35		+18 6		−25·6	
6·40		+31·6		−25·2	
7·42		+44·1		−25·1	
8·50	154°·1	+56·6	+55°·9	−24·3	−24°·56
9·43		+67·7		−24·3	
10·41		+79·7		−23·9	

equation derived from the mean of the first five days when the group was near the East Limb is

$$+0\cdot05x - 0\cdot94y - 24^{\circ}\cdot16 = z \quad . \quad . \quad . \quad (\alpha)$$

Similarly the last five days when the group was near the West Limb give

$$-0\cdot81x + 0\cdot47y - 24^{\circ}\cdot56 = z \quad . \quad . \quad . \quad (\beta)$$

The four middle days of observation when the group was near the Central Meridian would in like manner lead to the equation

$$-0\cdot82x - 0\cdot50y - 24\cdot92 = z \quad . \quad . \quad . \quad (\gamma)$$

In the earlier paper the equation

$$+0\cdot86x - 1\cdot41y = -0''\cdot40 \quad . \quad . \quad . \quad (\delta)$$

obtained by subtracting equation (β) from equation (α) was formed. Subtracting (γ) from the mean of (α) and (β) we find

$$+0\cdot44x + 0\cdot26y = -0''\cdot56 \quad . \quad . \quad . \quad (\epsilon)$$

The smallness of the coefficients of x and y in the equation (ϵ) as compared with those of the equation (δ) would make it of small weight, if the right-hand sides of the two equations arose entirely from errors of measurement and other errors of purely accidental character. But the equation (ϵ) has two advantages over the equation (δ). The orientation error of the photograph is entirely eliminated. This is important, as the same orientation error might persist for a series of spot-groups which occurred at the same time of the year. More important than this, the proper motion of each spot, so far as it is linear, is eliminated. In consequence of this, the right-hand sides of the equations similar to (ϵ), which for the sake of brevity will be alluded to hereafter as Central equations, have a much smaller accidental error than those similar to equation (δ), which will be termed Limb equations.

In the case, therefore, of all groups observed for 11 or more consecutive days, a second equation has been formed by subtracting the equation derived from the determinations of latitude made near the Central Meridian from the mean of the two equations given by those made near the two limbs respectively. Thus in a group observed for 11 consecutive days the first 4 and last 4 days were used to form the two Limb equations, the remaining 3 days, in the middle of the period of observation, to form the central equation. With groups observed for 12 days the 4 middle days were used for the Central equation; this left 4 days for each of the Limb equations, and necessitated the re-computation of the 12-day groups; for in the earlier paper, the mean of 5 days' observations had been taken for these groups at each limb. With groups of either 13 or 14 days' duration, the first 5 days and last 5 days gave the Limb equations, as in the earlier paper, and the remaining 3 or 4 days, as the case might be, afforded the Central equation.

The employment of observations made near the Central Meridian was not contemplated when the discussion was first undertaken, but it was perceived in the course of the work that such observations could yield information of value. Had they been used from the first, the labour involved would have been greatly lessened, and the method of reduction made more uniform by forming the Limb equations in all cases by taking the means of 3 observations at each end of the period of the visibility of a group, and deriving the Central equations from the mean of the remaining days. The weight of the equations would have been slightly greater if this procedure had been adopted.

In this connection a mistake in the earlier paper should be pointed out. On p. 573 of vol. lxxii. the sentence at the top of

the page should read:—"Thus y is obtained with a probable error of $\frac{\epsilon}{\sqrt{7}} = 0{\cdot}38\epsilon$, but x, from the way in which it is involved with z, with the much greater probable error, $0{\cdot}8\epsilon$. Thus as regards the position of the Sun's axis, four-fifths of the weight is concentrated in one of the normal equations, viz. that of y."

4. In Table XVII., which follows, are given the means of the quantities (irrespective of sign) on the right-hand sides of the equations formed in the three foregoing methods; *i.e.* from observations of spot-groups made near the Limbs only, from those in which observations made near the Centre were also used, and from Recurrent groups. As the quantities x and y to be determined from these equations of condition are very small, Table XVII. practically gives the mean discordances, and indicates the weights with which the separate groups should be combined. The first column indicates the class of observations concerned, the second

TABLE XVII.

Class of Observations.	Duration of Spot-groups.	No. of Groups.	Mean Discordances.	Means.
Limbs	8 days	327	±0°·562	
	9 „	288	0·670	
	10 „	293	0·734	
	11 „	313	0·704	
	12 „	524	0·645	
	13 „	434	0·623	
	14 „	33	0·596	
				±0·648
Central	11 days	313	±0·278	
	12 „	524	0·262	
	13 „	434	0·246	
	14 „	33	0·264	
				±0·262
Recurrent Groups		738	±0·971	
				±0·971

the length of time that the spot-groups were under observation, the third the total number of groups employed, and the fourth the mean discordances.

In the earlier paper the Limb groups were weighted according to the number of observations used in forming the mean longitudes and latitudes. Thus as 3 observations were taken together for the spot-groups of 8, 9, and 10 days' duration, all these were given the weight 3; the 11-day groups were given the weight 4, as the means were taken from 4 observations, and the weight 5 was given to spot-groups of 12, 13, and 14 days' duration. In the present paper the spot-groups of different duration, when only the Limb observations have been used, have been combined together with equal weights for each spot-group, since Table XVII.

shows that they are approximately of the same weight. The same applies also to the different durations when the Centre observations have been also used. The mean discordances of the residuals given by the three different methods are

$$\pm 0^{\circ}\cdot 648\,; \quad \pm 0^{\circ}\cdot 262\,; \quad \pm 0^{\circ}\cdot 971\,;$$

the relative weights are

$$2\cdot 38 \;:\; 14\cdot 57 \;:\; 1\cdot 06,$$

or approximately as

$$2 \;:\; 12 \;:\; 1.$$

These weights have been adopted in combining the equations resulting from the different methods in order to give the combined mean results in Table XIX.

5. Table XVIII. gives the numbers of groups of sun-spots of each kind used in forming the various equations. Those given in the ninth column, "Total observed at Limb," correspond to those given in the third column of Table XII. in the earlier paper, but the observations of 1912 have been included, and those of 1905 have been completely revised. In four cases, a spot-group observed at the turn of the year has been, more correctly, included with the year in which it passed the Central Meridian, instead of that in which it was first observed.

Table XIX. gives values for x and y obtained by combining the equations from the different spot-groups year by year, except for years near the sun-spot minima, when three or four years are taken together. The results are given, first for each of the three methods separately, and finally for the three combined. The second and sixth columns, headed "Limb," correspond therefore to the fifth and sixth columns of Table XII. The discordances from the earlier paper are due to the removal of the weighting in favour of the groups of longer duration, and to the change in the number of observations used in forming the means for the 12-day groups, much the most numerous of the seven classes.

The equations have also been combined together, not only for the separate years, for the three completed sun-spot cycles, and for the entire period, but also according to the phase of the cycle. The years 1881–2, 1892–3, and 1904–5 were allotted to the Increasing phase; 1883, 1894–5, and 1906–7 to Maximum; 1884–5, 1896–7, and 1908–9 to the Decreasing phase, and the remaining 22 years to Minimum.

An examination of Table XIX. shows that—

(i) The corrections to the adopted position of the Sun's axis, as indicated by the entire material collected, are very small; viz. $\Delta I = -4'\cdot 5$, $\Delta N \sin I = +0'\cdot 9$.

(ii) The agreement between the results of the three methods is fairly close.

(iii) There are wide discordances between the results for different individual years, the spots observed near the Limbs giving the smallest range, and the Recurrent groups the widest, as would

TABLE XVIII.

Numbers of Groups of Sun-spots employed.

Year.	Groups observed consecutively for							Total observed at		Recurrent Groups.	Total No. of Equations.
	8d	9d	10d	11d	12d	13d	14d	Limb.	Centre.		
1874–1877	12	14	12	17	12	8	0	75	37	26	138
1878–1880	7	5	5	13	16	5	0	51	34	19	104
1881	17	12	6	11	17	13	1	77	42	30	149
2	11	12	12	15	20	13	2	85	50	38	173
3	18	13	20	12	27	22	1	113	62	37	212
4	15	18	14	16	30	21	0	114	67	47	228
5	4	7	8	13	30	24	0	86	67	34	187
1886–1888	12	3	11	14	20	19	2	81	55	27	163
1889–1891	12	10	9	13	15	24	3	86	55	34	175
1892	12	20	18	19	31	29	3	132	82	43	257
3	24	29	20	25	42	32	2	174	101	62	337
4	28	21	17	14	35	38	4	157	91	47	295
5	17	14	17	17	30	26	1	122	74	33	229
6	12	9	12	8	14	10	1	66	33	17	116
7	6	5	4	9	12	15	0	51	36	19	106
1898–1900	13	9	12	11	19	13	1	78	44	19	141
1901–1903	7	8	6	10	7	13	0	51	30	12	93
1904	18	12	5	8	17	14	0	74	39	16	129
5	11	12	22	9	18	6	1	79	34	27	140
6	14	19	13	11	21	17	7	102	56	32	190
7	22	10	13	17	28	25	1	116	71	41	228
8	13	11	21	14	18	16	2	95	50	28	173
9	15	9	5	8	24	25	1	87	58	34	179
1910–1912	7	6	11	9	21	6	0	60	36	16	112
1878–1888	84	70	76	94	160	117	6	607	377	232	1216
1889–1900	124	117	109	116	198	187	15	866	516	274	1656
1901–1912	107	87	96	86	154	122	12	664	374	206	1244
1878–1912	315	274	281	296	512	426	33	2137	1267	712	4116
1874–1912	327	288	293	313	524	434	33	2212	1304	738	4254
Minimum	70	55	66	87	110	88	6	482	291	153	926
Increasing	93	97	83	87	145	107	9	621	348	216	1185
Maximum	99	77	80	71	141	128	14	610	354	190	1154
Decreasing	65	59	64	68	128	111	4	499	311	179	989

TABLE XIX.

Corrections to the Adopted Position of the Sun's Axis.

Period.	*x*.				*y*.			
	Limb.	Centre.	Recurrent.	Combined.	Limb.	Centre.	Recurrent.	Combined.
1874–1877	−10·0	− 1·0	−16·7	−8·4	+14·4	−16·6	+ 0·0	+ 7·0
1878–1880	− 0·8	−20·9	−11·5	−5·9	− 3·4	− 7·7	− 7·2	− 7·6
1881	+ 1·3	−31·8	+23·3	−0·9	− 9·0	+13·5	+ 1·9	− 3·7
2	+ 9·9	− 1·3	+ 5·3	+7·6	− 8·3	− 0·1	+ 2·5	− 5·2
3	− 0·6	+12·0	−28·8	−0·2	− 5·3	− 1·6	− 5·2	− 4·4
4	+11·5	− 1·7	+15·9	+9·7	+ 0·5	− 5·7	+12·7	+ 0·6
5	+14·1	− 8·3	+21·5	+9·8	− 7·6	+ 0·1	−12·6	− 7·3
1886–1888	− 3·7	− 5·6	+ 4·1	−2·5	− 7·5	− 2·4	−18·9	− 7·2
1889–1891	− 3·0	+ 7·9	+22·0	+2·8	− 4·0	−20·7	−21·7	− 9·6
1892	+ 5·9	+ 7·4	− 2·6	+5·7	− 8·7	− 9·5	+ 2·9	− 7·7
3	+ 1·5	+ 1·9	−17·7	− 1·0	− 9·9	− 4·2	− 1·2	− 7·5
4	+ 2·2	− 0·6	−10·4	+0·3	+ 0·8	− 2·9	+ 2·0	+ 0·0
5	+ 5·5	− 0·2	+11·8	+5·0	− 9·8	− 4·5	− 3·2	− 8·1
6	− 3·8	+10·0	− 9·5	−2·8	−11·7	+ 5·8	+ 8·7	− 6·9
7	+10·2	− 5·4	+ 9·7	+6·9	−12·7	+ 3·9	−11·9	− 6·9
1898–1900	− 3·3	−10·0	− 9·7	−5·1	−12·7	− 6·9	−12·5	−11·2
1901–1903	+ 5·3	+ 7·1	+28·0	+8·4	− 3·1	+11·6	+ 4·1	− 0·2
1904	+ 1·0	−16·7	+10·1	−2·4	+ 3·3	− 2·7	+20·2	+ 3·7
5	+ 2·4	+ 8·4	+ 2·0	+2·0	− 4·1	+ 9·5	−17·7	− 3·6
6	− 5·9	+14·5	+ 0·6	−1·0	− 5·3	− 8·9	−23·2	− 9·2
7	+ 0·3	− 8·6	− 2·2	−1·1	+ 8·7	−10·9	−20·8	− 0·3
8	− 8·5	− 1·3	+ 4·4	−5·3	− 5·9	+11·3	−16·3	− 3·1
9	−10·2	+ 1·4	− 2·0	−6·4	+ 9·2	−19·9	−22·2	− 2·8
1910–1912	− 0·4	+ 7·5	+ 6·6	+1·8	− 3·9	+ 6·1	− 3·9	− 1·2
1878–1888	+ 5·5	− 5·3	+ 5·3	+3·4	− 5·5	− 1·2	− 4·7	− 4·6
1889–1900	+ 2·1	+ 1·7	− 2·2	+1·6	− 7·6	− 5·6	− 3·0	− 6·7
1901–1912	− 2·8	+ 1·9	+ 0·5	−1·4	− 0·0	− 3·5	−14·3	− 2·6
1878–1912	+ 1·5	− 0·2	+ 1·0	+1·2	− 4·8	− 3·8	− 7·1	− 4·8
1874–1912	+ 1·2	− 0·2	+ 0·4	+0·9	− 4·2	− 4·1	− 6·9	− 4·5
Minimum	− 2·5	− 0·3	+ 0·5	−1·7	− 3·3	− 6·4	−12·9	− 5·0
Increasing	+ 3·7	− 2·3	− 1·3	+1·9	− 7·1	− 1·6	− 0·4	− 5·1
Maximum	+ 0·7	+ 3·5	− 5·4	+0·6	− 2·6	− 5·5	− 9·4	− 4·2
Decreasing	+ 2·3	− 1·3	+ 7·7	+2·3	− 3·3	− 3·2	− 7·3	− 3·7

naturally follow from the relative numbers of observations available for the three methods.

(iv) The irregular way in which these discordances are distributed indicates that they are due to irregular proper motions of spot-groups, large as compared with the errors in the adopted position of the axis.

(v) There is no sufficient evidence of any regular progressive motion of the axis during the period of observation.

(vi) Nor of any motion periodic with the sun-spot cycle.

6. To these conclusions may be added :—

(vii) There is no strongly marked evidence of any progressive difference in the determined position of the axis, depending on the latitude of the spot-groups from whence it is derived.

This appears from Table XX., the figures in which do indeed suggest a slightly larger correction to the inclination of the axis as derived from spots north of the equator than from those south of it; but the difference from the mean value is less than 2′. The difference is a little more marked if only spots with latitudes greater than 10° are considered, but it still remains under 3′. This range of difference is the same as that shown by the three complete sun-spot cycles, and by the four different phases of the cycle. Within this range, therefore, the position of the axis does not differ for different cycles, for different phases of the cycle, or for different zones.

TABLE XX.

Corrections to the Adopted Position of the Sun's Axis as determined from Spot-groups in Different Zones.

Zone.	*x*.				*y*.			
	Limb.	Central.	Re-current.	Com-bined.	Limb.	Central.	Re-current.	Com-bined.
	′	′	′	′	′	′	′	′
Above +18°	−0·2	+1·1	+1·7	+0·3	−5·8	−2·2	−9·8	−5·6
+18+14	+2·1	−4·9	+13·3	+1·6	−8·5	−7·6	−8·3	−8·2
+14+10	+4·5	−3·3	+4·0	+3·0	−3·9	−8·6	−17·5	−6·5
+10+6	−3·3	+0·3	−13·6	−3·8	−3·1	−7·6	−6·3	−4·6
+6+0	+8·8	−1·8	+11·7	+7·3	−7·6	−3·7	−4·0	−6·1
−0−6	−4·7	+0·9	−3·1	−3·6	+6·7	−9·1	−0·8	+2·1
−6−10	+0·9	+5·8	+7·7	+2·6	−8·6	−1·3	−4·1	−6·3
−10−14	+3·1	−9·7	−8·3	−0·7	−2·7	+0·1	−6·0	−2·3
−14−18	−1·6	+9·7	+0·1	+1·1	−3·9	+0·3	−3·5	−3·0
Below −18	+3·7	+0·1	−7·5	+1·6	−2·4	−5·3	−2·2	−3·0
Above +10	+2·4	−2·4	+5·7	+1·7	−5·8	−6·5	−12·5	−6·7
+10−10	−0·4	+1·9	+0·3	+0·1	−3·8	−5·1	−4·5	−4·2
Below −10	+1·7	+0·2	−5·0	+0·7	−3·1	−1·3	−3·6	−2·8
North	+1·8	−1·8	+2·8	+1·1	−5·4	−6·4	−11·3	−6·2
South	+0·8	+1·2	−1·9	+0·6	−3·1	−2·2	−3·5	−3·0
All spots	+1·2	−0·2	+0·4	+0·9	−4·2	−4·1	−6·9	−4·5

7. Spot-groups observed for 11 or more consecutive days are seen practically for a complete transit, from one limb to the other, and observations of them must be, in the mean, symmetrical with regard to the Central Meridian. This is not the case with groups of shorter duration, with the 8-, 9-, and 10-day groups. Of these some, which may be termed Eastern groups, are seen from the East limb onwards, but disappear before reaching the West limb; others, which may be termed Western groups, make their first appearance some distance from the East limb, but are followed up to the West limb; while others again are Central groups, forming and disappearing within the visible hemisphere. Since there is a systematic preponderance of the first of these three classes over the others, and of Central groups over the Western, the groups of short duration are not, on the whole, distributed quite symmetrically with respect to the Central Meridian, and hence the data they supply for the correction of the Sun's axis may be open to systematic error. The following table, however, does not indicate with certainty any correction due on this account. It is true that, when the groups of all the three durations are taken together, there appears to be a slight progression in the values of x and y, as we pass from the Eastern to the Central, and thence to the Western groups; but the discordances between the groups of different durations taken separately are too large to permit much significance to be attached to this circumstance.

TABLE XXI.

Class of Spot-group.	Duration of Spot-group.	x.	y.
Eastern	8 days	+ 5·7	− 0·4
	9 „	− 4·9	− 1·1
	10 „	+ 4·7	− 9·0
	8–10 „	+ 2·3	− 4·6
Central	8 days	+ 1·7	− 6·3
	9 „	− 0·3	− 0·7
	10 „	+ 1·6	+ 3·8
	8–10 „	+ 0·8	− 0·3
Western	8 days	+ 4·6	+11·3
	9 „	−11·4	− 2·1
	10 „	− 0·7	− 3·7
	8–10 „	− 1·8	+ 1·0
All positions	8 days	+ 4·3	+ 0·8
	9 „	− 4·6	− 1·2
	10 „	+ 2·4	− 3·9
	8–10 „	+ 0·8	− 1·8

TABLE XXII.

Numbers of Observations.

Zone.	Limb.				Recurrent Groups.			
	Minimum.	Increasing.	Maximum.	Decreasing.	Minimum.	Increasing.	Maximum.	Decreasing.
Above +18°	68	113	45	6	33	38	17	...
+18 +14	33	74	60	24	13	29	15	8
+14 +10	35	97	85	68	7	32	36	21
+10 + 6	44	42	71	60	7	10	19	23
+ 6 + 0	19	10	31	31	10	4	7	9
− 0 − 6	43	10	28	49	10	...	5	24
− 6 −10	74	29	64	79	25	11	17	30
−10 −14	68	53	79	81	22	16	25	28
−14 −18	36	75	85	74	14	31	26	28
Below −18	62	118	62	27	12	45	23	8
Above +10	136	284	190	98	53	99	68	29
+10 −10	180	91	194	219	52	25	48	86
Below −10	166	246	226	182	48	92	74	64
North	199	336	292	189	70	113	94	61
South	283	285	318	310	83	103	96	118
All spots	482	621	610	499	153	216	190	179

Zone.	1874 to 1888.	1889 to 1900.	1901 to 1912.	1874 to 1912.	1874 to 1888.	1889 to 1900.	1901 to 1912.	1874 to 1912.
Above +18°	71	102	59	232	34	37	17	88
+18 +14	53	86	52	191	21	26	18	65
+14 +10	84	107	94	285	24	38	34	96
+10 + 6	62	66	89	217	22	14	23	59
+ 6 + 0	25	41	25	91	10	11	9	30
− 0 − 6	45	42	43	130	14	11	14	39
− 6 −10	86	95	65	246	36	28	19	83
−10 −14	102	105	74	281	40	33	18	91
−14 −18	75	97	98	270	31	31	37	99
Below −18	79	125	65	269	26	45	17	88
Above +10	208	295	205	708	79	101	69	249
+10 −10	218	244	222	684	82	64	65	211
Below −10	256	327	237	820	97	109	72	278
North	295	402	319	1016	111	126	101	338
South	387	464	345	1196	147	148	105	400
All spots	682	866	664	2212	258	274	206	738

TABLE XXIII.

Mean Discordances in Latitude.

Zone.	Limb.				Recurrent Groups.			
	Minimum.	Increasing.	Maximum.	Decreasing.	Minimum.	Increasing.	Maximum.	Decreasing.
Above +18°	+ 5″5	+ 3″1	+ 5″1	− 8″4	+ 5″2	+16″0	+ 4″2	...
+18+14	−11·1	+ 0·3	− 3·8	− 8·1	−12·4	− 9·7	+ 9·2	+13·0
+14+10	− 4·1	−10·8	− 6·6	− 3·5	+18·2	−28·9	− 4·3	+ 3·2
+10+ 6	+ 1·0	− 4·1	− 1·8	− 9·7	−53·4	− 6·8	−21·5	+16·3
+ 6+ 0	− 1·1	−14·9	+ 1·4	−12·7	+ 5·3	−74·4	−14·0	− 2·4
− 0− 6	+ 0·7	+ 9·5	+ 4·4	+ 8·0	+16·6	...	−30·5	+25·8
− 6−10	− 1·7	+ 9·1	− 6·7	+ 5·6	− 5·2	+41·7	+30·1	− 5·8
−10−14	− 1·7	− 4·2	− 2·8	− 1·1	− 4·0	− 1·2	+ 2·0	+12·2
−14−18	+13·6	+ 1·6	− 5·6	+ 8·9	+35·9	+ 2·9	−21·1	+21·4
Below −18	− 4·9	−12·8	+ 8·3	− 2·2	− 4·0	+ 1·3	−10·4	−40·9
Above +10	− 1·0	− 2·4	− 2·9	− 4·9	+ 2·6	− 6·1	+ 0·8	+ 5·9
+10−10	− 0·4	+ 0·4	− 2·0	− 0·6	− 5·5	+ 3·8	− 3·1	+ 9·3
Below −10	+ 0·4	− 6·5	− 0·8	+ 2·8	+ 7·7	+ 1·4	−10·0	+ 9·6
North	− 0·5	− 3·0	− 2·2	− 7·7	− 2·6	− 8·5	− 4·8	+ 8·6
South	− 0·1	− 4·4	− 1·6	+ 4·3	+ 4·9	+ 5·7	− 4·0	+ 9·0
All spots	− 0·3	− 3·6	− 1·9	− 0·2	+ 1·4	− 1·7	− 4·4	+ 8·9

Zone.	1874 to 1888.	1889 to 1900.	1901 to 1912.	1874 to 1912.	1874 to 1888.	1889 to 1900.	1901 to 1912.	1874 to 1912.
Above +18°	+19″7	− 3″1	− 2″9	+ 3″9	− 1″0	+16″3	+16″6	+ 9″7
+18+14	+ 0·0	− 8·6	− 0·4	− 4·0	+ 0·1	−15·1	+10·6	− 3·1
+14+10	+ 1·3	−15·5	− 4·7	− 7·0	+10·1	− 0·2	−32·9	9·2
+10+ 6	− 0·2	−12·8	+ 0·2	− 3·8	−29·5	− 3·7	+ 9·8	− 8·0
+ 6+ 0	− 5·0	− 8·1	− 2·5	− 5·7	−13·7	−10·6	−12·1	−12·1
− 0− 6	+ 0·1	+ 3·4	+11·6	+ 5·0	+31·2	−10·3	+21·1	+16·3
− 6−10	+11·2	− 5·0	− 5·0	+ 0·6	− 1·3	+23·9	+ 2·3	+ 8·0
−10−14	+ 2·6	−10·7	+ 3·0	− 2·2	−14·8	+17·6	+16·3	+ 3·1
−14−18	+10·8	−10·9	+10·6	+ 2·9	+32·6	+12·5	−20·4	+ 6·5
Below −18	+ 6·4	−10·3	− 8·9	− 5·0	−20·1	+ 5·7	−17·2	− 6·4
Above +10	+ 7·3	− 9·2	− 3·1	− 2·6	+ 2·7	+ 2·0	− 9·4	− 1·0
+10−10	+ 3·8	− 6·2	+ 0·6	− 0·8	− 4·7	+ 6·1	+ 7·0	+ 2·2
Below −10	+ 6·2	−10·6	+ 2·9	− 1·4	− 1·1	+11·2	−10·4	+ 1·3
North	+ 4·6	− 9·7	− 2·1	− 3·2	− 5·2	+ 0·2	− 5·3	− 3·2
South	+ 6·5	− 8·2	+ 2·5	− 0·4	+ 2·0	+12·1	− 4·0	+ 4·1
All spots	+ 5·7	− 8·9	+ 0·3	− 1·4	− 1·1	+ 6·6	− 4·6	+ 0·8

The foregoing comparison as to the effect on the determination of the Sun's axis of the mean position of spot-groups of short duration—whether near the East limb, the Central Meridian, or the West limb respectively—was made soon after the completion of the earlier paper, and the same material was employed as for that. The result of the comparison appearing to be of a negative character, it did not appear worth while to go through the computations again in order to make the small alterations which the use of the very slightly fuller material of the present paper would introduce.

8. It will be remembered that in Carrington's determination of the position of the Sun's axis he found an apparent tendency of spots to move northward at the rate of 90″ in the time during which the Sun rotates through 30° (*Observations of the Spots on the Sun*, p. 242). This he regarded as "a fictitious and non-real result," but it seemed worth while to inquire if the Greenwich measures confirmed it or otherwise; and in particular—having in view the well-known change in the latitude-distribution of sun-spots with the progress of the solar cycle—whether any tendency to move towards the equator or from it, characterised the spots of any particular zone, or at any particular phase of the sun-spot cycle. For this purpose the observed differences of latitude, *i.e.* the quantities on the right-hand side of the equations of condition, in the cases of the Limb equations and of those from the Recurrent groups, have been grouped in ten zones, according to the latitude of the spot-group concerned. The Central equations having been cleared, by the way in which they have been formed, from the effects of a steady uniform drift in any one given direction, have not been treated in this manner.

Tables XXII. and XXIII. give respectively the numbers of observations, and the mean discordances in latitude for the different zones, both for the three cycles of the sun-spot activity over which the observations extend, and for the several phases of the progress of the cycle. As in the earlier paper, motion towards the N. Pole is considered positive.

It will be seen that the observations do not establish the reality of a common drift of the sun-spots to the extent that the figures derived by Carrington might be taken to indicate. The average interval corresponding to the mean discordances from the Limb equations is 7 days, in which time the Sun rotates through 100°; and the average interval for the Recurrent groups is 18 days, or more than $2\frac{1}{2}$ times as long. A movement of 1′·5 in latitude for a rotation of 30° would show as 5′ from the Limb equations, and as 13′ from those given by the Recurrent groups. But instead of a northward movement of this amount the final result from the Recurrent groups is almost *nil*, and from the Limb spots is a very small movement southward, while the individual figures given in the tables show a wide range. In other words, though in limited areas of the Sun's surface groups of spots are often observed to have a "set" in one direction or the other that

may last for some time, these tend in the long run to balance one another.

For the Limb equations, the two columns "1874 to 1888" and "1889 to 1900" are in somewhat striking contrast, and suggest that the photographs of both periods were generally affected by a sensible orientation error, but of different signs in the two periods. If this were the case, the Recurrent groups should reverse the effect, for orientation error will affect the differences in latitude from the Recurrent groups to the same amount as it will those from the spot-groups observed near the Limb, but with the opposite sign. The Recurrent groups are exposed to accidental errors of too large an amount, and the numbers of groups available in each zone in the separate periods are too few for the actual figures to have much significance; but so far as they go, when taken together with those from the Limb equations, they confirm the conclusion that small outstanding orientation errors gave a slight apparent northward motion to the spots in their transit across the disc in the period 1874 to 1888, but a southward motion in the period 1889 to 1900. The third period, 1901 to 1912, appears to have been practically free from any general error of this nature.

But though there is no satisfactory indication of any general drift of all spots in the same direction, the column "1874 to 1912" does suggest a slight tendency for spots in the outer zones of either hemisphere to move towards the pole of that hemisphere, while the spots in all other zones tend on the whole to move towards the equator. This appears to be shown both by the spots near the Limb and by the Recurrent groups.

9. In conclusion, the results obtained may be summarised:—

(1) Small corrections to Carrington's elements appear to be indicated; viz.

$$\Delta I = -4''.5 \qquad \Delta N \sin I = +0''.9,$$

giving the true position of the solar equator, for the epoch 1850·0, as

$$I = 7^\circ\ 10''.5 \qquad N = 73^\circ\ 46''.8.$$

(2) The final corrections, as deduced by the "Limb" and "Central" methods respectively, are in close accord; showing that they are substantially free from systematic errors due either to defective orientation of the photographs, or to a general drift of the spot-groups.

(3) The discordances between the corrections deduced from short periods, such as a single year, or from spot-groups in restricted zones, are due to the large proper motions of individual groups. These last show a certain tendency to run into sets, but do not persist over long periods of time.

(4) A slight general tendency is suggested for high-latitude spots in either hemisphere to move away from the equator, but for those in lower latitudes to move towards it.

Royal Observatory, Greenwich:
1913 June 13.

Mean Areas and Heliographic Latitudes of Sun-spots in the Year 1912.

(*Communicated by the Astronomer Royal.*)

The results here given are in continuation of those printed in the *Monthly Notices*, vol. lxxiii. p. 47, and are deduced from the measurements of photographs taken at the Royal Observatories of Greenwich and of the Cape, and in India at the Kodaikánal Observatory and at Dehra Dûn, North-West Provinces.

Table I. gives the mean daily area of umbræ, whole spots, and faculæ for each synodic rotation of the Sun in the year 1912, and Table II. gives the same particulars for each of the entire years 1901 to 1912 inclusive. The areas are given in two forms: first, projected areas, that is to say, as seen and measured on the photographs, these being expressed as millionths of the Sun's apparent disc; and next, areas as corrected for foreshortening, the areas in this case being expressed in millionths of the Sun's visible hemisphere.

Table III. exhibits for each rotation in the year 1912 the mean daily area of the whole spots (corrected for foreshortening), and the mean heliographic latitude of the spotted areas for spots north and for spots south of the equator, together with the mean heliographic latitude of the entire spotted area and the mean distance from the equator of all spots; and Table IV. gives the same information for each year as a whole for the years 1901 to 1912 inclusive. Tables II. and IV. are thus in continuation of the similar tables for the years 1874 to 1888 on

Table I.

No. of Rotation.	Date of Commencement of each Rotation.		No. of Days on which Photographs were taken.	Mean of Daily Areas.					
				Projected.			Corrected for Foreshortening.		
				Umbræ.	Whole Spots.	Faculæ.	Umbræ.	Whole Spots.	Faculæ.
	1912.	d							
780	Jan.	13·42	27	0	0	221	0	0	232
781	Feb.	9·76	27	0	4	169	1	7	189
782	Mar.	8·09	27	16	88	158	11	59	172
783	Apr.	4·39	28	23	109	189	19	97	199
784	May	1·65	27	3	27	258	2	17	243
785	May	28·87	27	17	95	376	12	77	387
786	June	25·07	26	8	45	223	6	33	238
787	July	22·27	27	0	0	109	0	0	115
788	Aug.	18·50	28	4	24	182	3	16	191
789	Sept.	14·75	27	29	139	257	20	98	283
790	Oct.	12·03	27	0	1	187	0	1	195
791	Nov.	8·33	28	1	5	102	1	3	104
792	Dec.	5·64	27	27	143	204	18	96	201

TABLE II.

Year.	No. of Days on which Photographs were taken.	Mean of Daily Areas. Projected. Umbræ.	Whole Spots.	Faculæ.	Corrected for Foreshortening. Umbræ.	Whole Spots.	Faculæ.
1901	359	14	41	23	9	29	29
1902	349	14	86	163	10	62	178
1903	350	67	434	875	51	340	970
1904	363	93	653	1639	67	488	1761
1905	364	230	1637	2433	163	1191	2612
1906	364	158	1047	2207	114	778	2320
1907	365	221	1453	1859	160	1082	1999
1908	366	148	952	2000	106	697	2098
1909	364	171	941	1303	124	692	1353
1910	365	67	357	984	49	264	971
1911	364	16	88	458	12	64	459
1912	365	10	50	201	7	37	210

TABLE III.

No. of Rotation.	Date of Commencement of each Rotation.	No. of Days on which Photographs were taken.	Spots NORTH of the Equator. Mean of Daily Areas.	Mean Heliographic Latitude.	Spots SOUTH of the Equator. Mean of Daily Areas.	Mean Heliographic Latitude.	Mean Heliographic Latitude of entire Spotted Area.	Mean Distance from Equator of all Spots.
	1912. d			°		°	°	°
780	Jan. 13·42	27	0·0	...	0·0	...	...	...
781	Feb. 9·76	27	0·0	...	7·1	11·91	−11·91	11·91
782	Mar. 8·09	27	0·2	11·90	58·8	12·47	−12·40	12·47
783	Apr. 4·39	28	0·6	12·30	95·9	10·83	−10·67	10·84
784	May 1·65	27	0·5	11·23	16·9	10·45	− 9·85	10·48
785	May 28·87	27	0·8	9·90	75·7	6·95	− 6·76	6·99
786	June 25·07	26	0·0	...	32·9	5·53	− 5·53	5·53
787	July 22·27	27	0·0	...	0·0	...	...	...
788	Aug. 18·50	28	0·0	...	16·3	6·08	− 6·08	6·08
789	Sept. 14·75	27	0·0	...	97·6	8·14	− 8·14	8·14
790	Oct. 12·03	27	0·0	...	1·4	4·77	− 4·77	4·77
791	Nov. 8·33	28	0·0	...	3·1	2·63	− 2·63	2·63
792	Dec. 5·64	27	11·1	23·72	85·1	1·89	+ 1·06	4·40

TABLE IV.

Year.	No. of Days on which Photographs were taken.	Spots NORTH of the Equator.		Spots SOUTH of the Equator.		Mean Heliographic Latitude of entire Spotted Area.	Mean Distance from Equator of all Spots.
		Mean of Daily Areas.	Mean Heliographic Latitude.	Mean of Daily Areas.	Mean Heliographic Latitude.		
1901	359	22	8°·59	7	16°·27	+2°·82	10°·37
1902	349	42	18·81	21	15·29	+7·48	17·64
1903	350	132	18·11	208	21·15	−5·85	19·94
1904	363	268	16·33	220	16·88	+1·37	16·57
1905	364	750	11·66	440	15·55	+1·60	13·10
1906	364	539	13·98	239	14·01	+5·38	13·99
1907	365	488	10·12	593	13·77	−2·98	12·12
1908	366	316	10·42	381	10·34	−0·92	10·38
1909	364	299	9·45	393	9·92	−1·55	9·71
1910	365	66	8·55	198	11·18	−6·27	10·53
1911	364	17	7·89	47	5·99	−2·30	6·49
1912	365	1	20·53	36	7·78	−7·18	8·06

pp. 381 and 382 of vol. xlix. of the *Monthly Notices*, and for the years 1889 to 1902 on pp. 465 and 466 of vol. lxiii., and for the years 1901 to 1911 on pp. 48 and 49 of vol. lxxiii.

The rotations in Table I. and Table III. are numbered in continuation of Carrington's series (*Observations of Solar Spots made at Redhill*, by R. C. Carrington, F.R.S.), No. 1 being the rotation commencing 1853 November 9. The assumed prime meridian is that which passed through the ascending node at mean noon of 1854 January 1, and the assumed period of the Sun's sidereal rotation is 25·38 days. The dates of the commencement of the rotations are given in Greenwich civil time, reckoning from mean midnight.

The principal features of the record for 1912 are:—

1. The further continuance of the rapid decline in the mean daily spotted area that began in 1910. Thus in 1912 the mean daily spotted area was only 37 millionths of the Sun's visible hemisphere as against 264 in 1910, and 64 in 1911. Comparing with the years of minimum in the three preceding cycles, which gave respectively: 1878 an area of 22; 1889 of 78; and 1901 of 29, the year 1912 would appear to have nearly fallen to the minimum level, has indeed fallen lower than the minimum of 1889. Since the current year (1913) has shown up to the present date (September 12) a much feebler condition of spot-activity even than 1912, it is clear that the sun-spot minimum now in progress is likely to prove unusually low and prolonged. Minima of this character in the past have generally been followed by a slow recovery leading up to a low maximum.

2. An examination of the results for the separate rotations shows that the year 1912 was distinguished by five slight revivals of activity, in March, April, June, September–October, and

December respectively; for the rest of the year the Sun was almost entirely free from spots.

3. The areas of the faculæ have, as usual, varied much less than those of the spots, but the falling off from 1911 is even more marked; the figure for 1912 being 46 per cent. of that for 1911 for the faculæ, and 58 per cent. for the spots. Comparing the mean daily area obtained for the faculæ in 1912—viz. 201—with the results obtained in the three preceding cycles, a still further decline is to be expected before the actual minimum is reached.

Year.	Mean Daily Area.	Year.	Mean Daily Area.	Year of Minimum.	Mean Daily Area.
1876	257	1877	162	1878	67
1887	301	1888	239	1889	131
1899	337	1900	180	1901	29

The state of the faculæ would suggest that the minimum will fall very late in 1913 or, more probably, in 1914.

4. Comparing the two hemispheres, the northern was absolutely free from spots during eight rotations, and almost free during four. The only period when there was any noteworthy activity north of the equator was in December. But the two groups of spots forming in the northern hemisphere during December were both in high latitudes, viz. $+20^\circ$ and $+26^\circ$ respectively. They are, therefore, indications of the commencement of a new period of activity. The southern hemisphere showed no great change in its characteristics from 1911. It was completely free from spots in two rotations, and nearly so in three, and the mean daily spotted area was 77 per cent. of its value for 1911.

5. The mean distance from the equator of all spots showed a distinct rise as compared with 1911, but did not reach the level of 1910. This is the case whether the high latitude northern spots of December be included or excluded; in the former case the distance from the equator comes out as $8^\circ{\cdot}06$, in the latter as $7^\circ{\cdot}97$, the results for 1910 and 1911 being $10^\circ{\cdot}53$ and $6^\circ{\cdot}49$ respectively.

6. The Sun was free from spots on 246 days in 1912, as compared with 183 in 1911.

7. In the distribution of spots in latitude, the disturbances in the southern hemisphere have ranged from 1° to 15°; and in the northern from 9° to 14° in the spring of the year, besides the two high-latitude groups already noted as observed during December.

8. The number of separate groups observed in 1912 was 39 instead of 62 as in 1911; or, expressed as a percentage of that year, it would stand as 63. This is a slightly smaller decline than that in area, so that the proportion of small and short-lived spots still continues to increase.

9. Of the 39 separate groups of the year, 32 were in the southern, and only 7 were in the northern hemisphere; or ex-

pressed in percentages, the proportion was 18 for the north to 82 for the south; this is the highest yet shown in the present cycle by the southern hemisphere, which has now been the more disturbed of the two for six years in succession.

Royal Observatory, Greenwich:
1913 *September* 12.

The Longitude of the Moon from 1750 *to* 1910.* (Plate 22.)

By Ernest W. Brown, M.A., Sc.D., F.R.S.

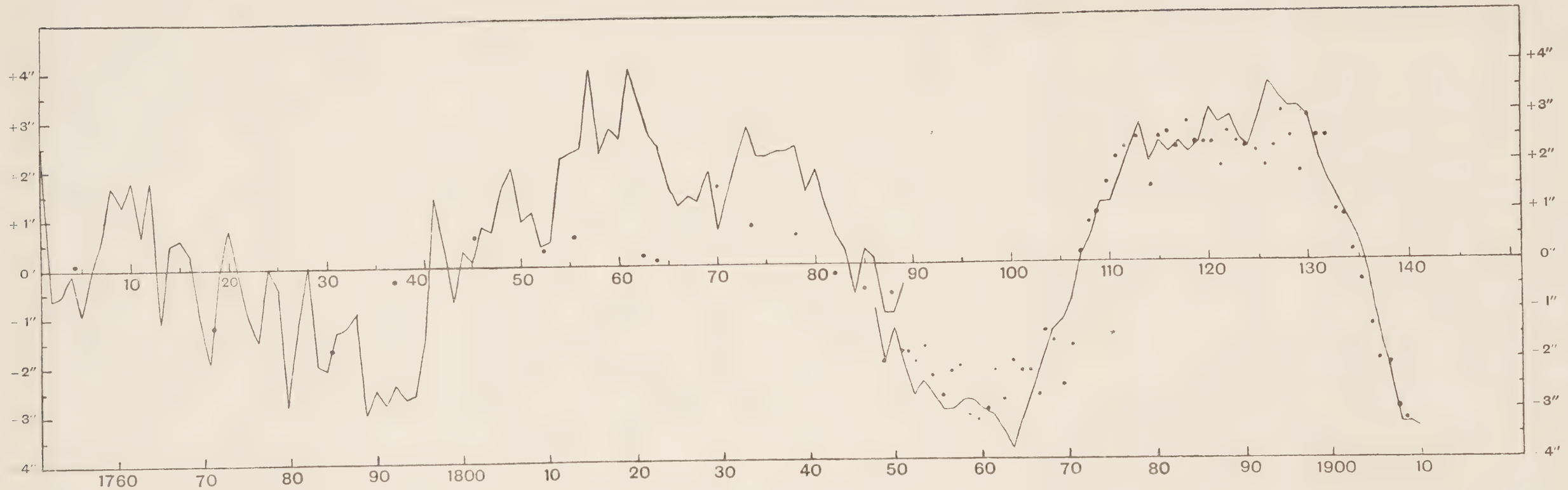
1. The original object of the work embodied in this paper was the determination of values for the mean motion and mean longitude at epoch of the Moon for use in the new tables now under construction for its motion. The actual values to be adopted are of less importance than a knowledge of what those values represent and whence they are derived. With this in view, it was necessary that the observations to be used should be compared with the theory which is reduced to numbers in the tables, and that any differences between this theory and the observations should be tabulated or expressed by a definite empirical formula.

It is not probable that the observed places of the Moon over a long interval can be directly compared with the new tables, and yet it is necessary that some form of comparison be made. The Greenwich observations for the past 160 years present advantages for this purpose which much outweigh the objection to the use of material gathered from one source only. They are continuous, and have been reduced on a consistent plan, which renders their correction and comparison a matter of far greater ease and certainty than observations gathered from a variety of sources. And further corrections in the future can be applied easily.

2. The greater part of the labour of collecting this material and comparing it with modern theory has been done by Dr. P. H. Cowell in a series of papers in the *Monthly Notices* from 1903 to 1905. In order that the tabular places should represent my theory it is necessary to correct them further for planetary and other terms whose values were not available at the time Dr. Cowell undertook the work. This refers mainly to the terms of periods longer than a year: those of short period (a year or less) have but little influence on the mean longitude when we take means over an interval of the order of a year, and, in any case, Dr. Cowell has inserted the most important corrections of these.

I have attempted to complete the comparison by adding the

* This paper was originally presented to the Society and read on April 11, under the title "A discussion of the empirical terms in the Moon's longitude." Just after it was sent off, Professor Newcomb's long-expected memoir on the occultations appeared. A study of his work showed that it was possible to improve further the comparison of theory with observation, and several additions have, in consequence, been made to the original paper. The latter contained an analysis of the differences between theory and observation into a periodic formula; this is now omitted for reasons stated below.



E. W. Brown.—Motion of the Moon 1750–1910 from Greenwich Meridian Observations and Occultations.

tabular corrections necessary in order to reduce the adopted theory to that which I have published in the *Memoirs* of the Society. Only those terms are considered which can sensibly affect the means taken over Cowell's period of 414 days. A complete comparison, for this purpose, was made between Airy's tabular theory, Hansen's tabular theory, Cowell's corrections to them, and my theory, and the additions which are made here are the result of this comparison. It should be stated that, while Cowell mentioned the most important of the alterations which were necessary in this connection, he did not apply them to the mean errors which are here copied from one of his papers. I have also investigated the observational corrections; these have been rather troublesome, and I have only been able to use those for change of equinox and conversion of time. The last part of the work has been mainly done by Mr. H. B. Hedrick, who has deduced the values and tabulated them for use.

3. The recent appearance of Newcomb's last memoir on the occultations* has made it possible to give a fairly complete comparison of the Greenwich observations with the occultations derived from a variety of sources. Newcomb uses Hansen's theory, but adds the latest values of the solar and planetary terms, so that little correction is necessary in order to reduce it to my theory. I have compared the two theories and have corrected his results accordingly.

Within the whole period of 160 years covered by the occultations and the Greenwich meridian observations, the agreement between the two can be considered complete so far as it is possible to determine the terms in the mean longitude of the form $a + bt + ct^2$ from this interval. The minor differences are discussed below. One defect in this comparison should be pointed out. In the Airy and Hansen periods of the Greenwich observations there is an overlapping interval of five years, 1847–51, with an average difference of 1″·0. The same observations are used in this period, and the difference in the reductions is fully discussed by Airy in his *Reduction of the Greenwich Observations of the Moon from* 1830–51. I have attempted to take into account all these differences, but have not succeeded in making a complete agreement between the residuals as deduced from the corrected results of Airy and Hansen. The mean for the five years deduced from the occultations agrees closely with that derived from the corrected results from Hansen's tables.

The diagram (Plate 22) shows the reality of the larger fluctuations in the Moon's motion; the agreement, especially since 1845, between the occultations and meridian observations is closer than might have been expected with the known sources of systematic errors of observation.

4. An extensive analysis, only incidental reference to which is made below, has been carried out in order to try and express these

* *Astronomical Papers for the Use of the American Ephemeris*, vol. ix., Pt. 1.

fluctuations by means of a harmonic formula. It is omitted on account of one unsatisfactory feature. Four terms whose periods seemed to be related to one another gave a good representation of the errors up to 1901, but when (later) the observations to 1910 were included, the formula gradually ran off to a difference of nearly 2″, no such gradual change having previously appeared. A later attempt with these observations included has not hitherto been completely successful. But if the residuals can be represented in such a manner at all, it seems clear that there must be at least three terms with periods between 75 and 35 years, and coefficients between 3″·0 and 0″·5.

5. For the purposes of the lunar tables, the chief result is contained in the discussion of the mean longitude, the secular acceleration, and the great empirical term. I conclude from the discussion that two courses are open. One is the adoption of the values of the secular acceleration, the mean longitude at epoch, the mean motion and the great empirical term as found by Newcomb from a combination of ancient eclipses and modern observations. The other is the adoption of the theoretical value of the secular acceleration and the change of the other quantities so that the modern observations are still satisfied. Nearly a century must elapse before a distinction between the resulting values of the constants can be fully determined. If no other empirical terms be included and the Moon continues to move as it has done for the past century and a half, we must still expect differences between observation and the tables, which will at certain periods amount to some four seconds of arc in longitude whichever of the two courses be adopted. This refers to means taken over periods comparable with a year. I believe that sensible fluctuations with periods comparable with a month also exist; if this proves to be the case, a particular phenomenon like an eclipse may show a residual difference of even greater magnitude quite apart from any error of observation.

The Tabular and Observational Corrections.

6. The residuals given by Dr. Cowell* are derived from two principal sources: the Greenwich observations from 1750 to 1851, reduced by Airy and compared with tabular places also constructed by Airy; and the Greenwich observations from 1847 to 1901, compared with tabular places derived from Hansen's tables of the Moon. To the latter I have added the residuals to 1910, deriving them from the volumes of Greenwich observations.

The Airy series was corrected by Cowell † (i) for the larger differences between the short-period (a year or less) terms used by Airy and modern values derived from theory; (ii) for the difference between the Airy (Damoiseau) and the Hansen-Newcomb mean longitude; (iii) by the addition of the empirical 273-year term now used in the *Nautical Almanac*.

* *M.N.*, vol. lxv. (1904), pp. 38, 39.
† *Ibid.*, pp. 36, 37.

The Hansen series he corrected * for certain short-period terms only, with the exception that the earlier part of the series was corrected by the elimination of the wrong Venus term which had been used in the tabular places, by the addition of the empirical term just mentioned, and by the change in the mean longitude introduced by Newcomb. So far, then, as these terms are concerned, the material was made homogeneous. To the observations from 1902–1910 the short-period corrections of Dr. Cowell have not been applied: no sensible errors will be caused in the means taken over 414 days by this omission.

7. I have further corrected the tabular places as follows:—(*a*) the two series for the planetary terms omitted by Airy and Hansen; (*b*) the Airy series for certain solar and planetary terms and for the terms due to the ellipticity of the Earth, to reduce them to the values used in my theory; (*c*) the Hansen series for similar sets of terms; (*d*) both series for a change to the value of the great Venus term deduced in my theory, for a change to the latest value found by Newcomb for the great empirical term, for three planetary terms of very long period, and for a change to the value of the mean longitude deduced by Newcomb in the paper to which reference has already been made. Under (*b*) and (*c*) are included also the time and equinox changes necessary to reduce the observations to Newcomb's solar tables and his fundamental catalogue of stars respectively.

The separation into these four classes has no theoretical significance: it depended only on conveniences of tabulation. The class (*a*) had already been tabulated from 1800 for the new tables; in class (*d*) the periods are very long; (*b*) and (*c*) are mainly corrections peculiar to the Airy and Hansen series respectively. The detailed exhibit of the various corrections applied is given in the following pages. The residuals thus corrected should therefore represent the difference between theory and observation for the past 160 years, so far as terms which can affect the means taken over 414 days are concerned.

The period of 414 days (accurately $414^{d}\cdot02$) is that used by Cowell. It has no special significance here, but I adopted it without hesitation since he had given the means of the residuals over this interval, and no extra work in the tabulation of the additional terms was caused by its use.

The symbol P is used to denote this period.

8. *The Correction* (*a*).—In the list given below, many of the terms are very small and would not have been included if they had not already been tabulated from 1800 for the purposes of the new tables: it was easier to complete the tabulation with them than without them. A few of them are not real corrections, but the maximum sum of these cannot amount to $0''\cdot1$. The notation is that of my lunar theory (*Mem. R.A.S.*, vol. lix.).

The results are contained in the second columns of Tables I., II.

* *M.N.*, vol. lxiv. pp. 571–3.

+ 0″·284 sin (3J − 2T + 2*l* − 2D + 172°·5)	+ ″·018 sin (8M − 6T + 2*l* − 2D + 244°)
+ ·282 sin (264°·0 − w_3)	+ ·017 sin (5T − 6M − 2*l* + 2D + 331°)
+ ·240 sin (2T − 2J − 2*l* + 2D + 0°·1)	+ ·017 sin (D − F + 2M + 165°)
+ ·237 sin (8V − 13T + 226°·1)	+ ·013 sin (15V − 12T − D + 278°)
+ ·126 sin (20V − 21T + *l* − 2D + 267°)	+ ·013 sin (23V − 25T − D + 350°)
+ ·108 sin (26V − 29T − *l* + 68°)	+ ·011 sin (*l* − D + 4T − 3V + 273°)
+ ·075 sin (2D − *l* + T − 3Q + 105°)	+ ·010 sin (3*l* − 2D + 24T − 24V)
+ ·073 sin (3V − 3T − 2*l* + 2D)	+ ·008 sin (18V − 17T + F − D − *l* + 10
+ ·062 sin (8T − 6V + 2*l* − 2D + 17°·4)	+ ·006 sin (6T − 11M + 205°)
+ ·054 sin (5V − 6T + 2D − 2F + 270°)	+ ·006 sin (7T − 13M + 161°)
+ ·040 sin (119°t_c + 152°)	+ ·006 sin (23V − 24T − *l* + 268°)
+ ·038 sin (2w_3 + 2°·8t_c + 180°)	+ ·006 sin (2J + w_3 + 168°)
+ ·033 sin (12V − 8T + *l* − 2D + 237°)	+ ·005 sin (J + w_3 + 45°)
+ ·030 sin (*l* + 21T − 21V)	+ ·004 sin (17M − 9T + 63°)
+ ·026 sin (8T − 15M + 137°)	+ ·003 sin (Q − 4T + 239°)
+ ·025 sin (2F − 2D − 2*l*′)	+ ·003 sin (24T − 23V + F + 285°)
+ ·025 sin (13T − 15V + 2D − *l* + 151°)	+ ·003 sin (4Q − 5T + *l* − 2D + 67°)
+ ·021 sin (S + 273°)	+ ·003 sin (4Q − 3T + *l* − 2F + 113°)
+ ·003 sin (2S + 297°)	+ ·003 sin (17V − 16T − 2D + *l* + 287°
+ ·019 sin (5T − 3V + w_3 + 216°)	+ ·003 sin (4J − 2T + 2*l* − 2D + 163°)

9. *The Corrections (b) to the Airy Series.* The list is :

− 0″·59 sin (*l* − *l*′ − D)	+ 0″·09 sin (4M − 2T + 245°)
+ ·26 sin 2(*l* − *l*′ − D)	+ ·04 sin (3M − 2T + 212°)
− ·20 sin (2*l* − 2F)	+ ·09 sin (J + 290°)
+ ·04 sin (2T − V + 82°)	+ ·51 sin ☊
+ ·05 sin (3T − 2V + 273°)	+ ·42t_c
+ ·09 sin (5T − 3V + 199°)	+ time correction
+ ·33 sin (2M − T + 224°)	+ equinox correction

Of these, the last four alone require special mention. The term, argument ☊, is an addition to the term used by Airy for the effect in ecliptic longitude due to the figure of the Earth. Airy has a coefficient + 6″·60; I use + 7″·11, corresponding to an ellipticity 1/297. The planetary part with this argument has already been included in (*a*) : Airy has no such part.

Some of the terms here and elsewhere are of such short period that we cannot properly use the theoretical value at the middle of

a period P to represent the mean value over the whole period. Whenever necessary, the coefficient was then multiplied by $\sin\theta/\theta$, where θ is the angle described by the argument in $\frac{1}{2}$P. In certain cases where a term had been already tabulated, the mean value was obtained by the use of the formula,

$$\frac{1}{n}\sum_{r=0}^{r=n-1}\sin(\alpha+r\beta)=\frac{\cos(\alpha-\frac{1}{2}\beta)-\cos(\alpha-\frac{1}{2}\beta+n\beta)}{2n\sin\frac{1}{2}\beta}.$$

If β be sufficiently small, $2n\sin\frac{1}{2}\beta$ is the angle described by the argument in the period P. Hence the rule: Choose the values of the term a quarter of its period in advance of the dates at which each period P begins, and divide each value by the angle described in P; the first differences of the table thus obtained will be the required mean values.

In correcting the tab.–obs. means, it was assumed that the observations were sufficiently equably distributed over each period P.

The term $0''.42t_c$, where t_c is the number of centuries from 1800, is a correction which Dr. Cowell * pointed out as necessary to complete the reduction of Airy's mean longitude to that of the Hansen-Newcomb system now used in the *Nautical Almanac*. I have verified the comparison.

10. The only systematic corrections to the observations which were found to be capable of affecting the means over 414 days were those due to the conversion of sidereal to mean solar time and to the position of the equinox. It was considered best to reduce these to Newcomb's solar tables and to his catalogue of fundamental stars, respectively. Mr. H. B. Hedrick writes as follows on these questions:

"*Time correction.*—Airy, in his reduction of the Greenwich Observations of the Moon from 1750 to 1851, computed the 'Mean Time of Observation,' from the 'Observed Right Ascension at Transit.' For that part of the reduction from Sidereal to Mean Solar time depending on the Mean Sun, he used, from 1750 to 1835, a set of tables given in his *Reduction of the Observations of the Moon made at Greenwich, from* 1750 *to* 1830, vol. i., on pp. lxvii–lxx, and in his *Reduction of the Observations of the Moon*, 1831 *to* 1851, on p. xii. After 1835, that is from 1836 to 1851, he used the data in the *Nautical Almanac*.

"The tables given by Airy agree substantially with the position and motion of the Mean Sun as given by Delambre in his Solar tables, 1806, and in Carlini's first Solar tables, 1810.

"In the *Nautical Almanac* after 1835, Carlini's new Solar tables, Milan 1832, were used. In the latter Carlini included Bessel's correction to Delambre's Mean Longitude of the Sun, which is stated by Bessel in the *Astr. Nach.*, No. 133, S. 266, to be—

$$\text{"Bessel} - \text{Delambre} = +2''.65 + 14''.45t_c.$$

* *M.N.*, vol. lxv. (1905), p. 272.

"Newcomb's correction to Bessel's Mean Longitude of the Sun is—

$$\text{"Newcomb} - \text{Bessel} = +4''\cdot75 + 5''\cdot39t_c - 0''\cdot132t_c^2,$$

and so Newcomb's correction to Delambre's Mean Longitude of the Sun is—

$$\text{"Newcomb} - \text{Delambre} = +7''\cdot40 + 19''\cdot84t_c - 0''\cdot132t_c^2,$$

where t_c stands, in all cases, for Julian centuries from 1800.

"This time correction may be converted into the form of a correction to the 'Obs. place' by the formula—

"1^s Change in Mean time of Obs. $= -0''\cdot55$ Change in Obs. place.

"*Equinox correction.*—In the Appendix iii. to the *Washington Observations* for 1870, Newcomb gives the following correction to Bessel's Regiomontana on account of equinox—

"Newcomb – Bessel's Regiomontana $= +0^s\cdot016 - {}^s\cdot009t_c$ from 1820.

"The following table gives the result of these two corrections:

Date.	Change in Mean Time of Obs.	Corres. Corr. to Obs. Place.	Equi. Corr. to Obs. Place.
	s	"	"
1750	+0·17	−0·09	+0·33
1760	+0·04	−0·02	+0·32
1770	−0·10	+0·05	+0·31
1780	−0·23	+0·13	+0·30
1790	−0·36	+0·20	+0·28
1800	−0·49	+0·27	+0·27
1810	−0·62	+0·34	+0·26
1820	−0·76	+0·42	+0·24
1830	−0·89	+0·49	+0·22
1835	−0·96	+0·53	...
1836	−0·45	+0·25	...
1840	−0·46	+0·25	+0·21
1850	−0·50	+0·27	+0·20"

The corrections contained in the two latter columns of this table were interpolated and inserted (with changed signs since tabular-observed errors are used in all cases) with the other corrections, noted above, in column (*b*) of Table I.

11. *The Corrections* (*c*) *to the Hansen Series.* The list is:

$+0''\cdot49 \sin(l - l' - D)$
$- \cdot03 \sin 2(l' - F + D)$
$- \cdot28 \sin(T - V)$
$+ \cdot35 \sin(3T - 2V + 273^\circ)$
$+ \cdot09 \sin(5T - 3V + 199^\circ)$
$+0''\cdot09 \sin(J + 290^\circ)$
$+1''\cdot22 \sin(\Omega + 325^\circ)$
+ time correction
+ equinox correction

The first six are corrections to Hansen's planetary and solar terms, terms not taken into the corrections (*a*), nor present in Hansen's tabular longitude, and terms present in (*a*), and also in Hansen's tables, which must therefore be subtracted here.

The term, with argument $\Omega + 325°$, is the combined correction of the ellipticity and planetary terms which have the node as argument. Hansen's tabular *nz* and *s* contain the terms,

$$nz = +7''\cdot760 \sin(-\Omega + 184°\cdot42') + 0''\cdot256 \cos g \sin(-\Omega),$$
$$s = +8''\cdot764 \sin(f + \omega + \Omega + 169°\cdot51').$$

The terms in the true longitude on the ecliptic which arise from these are

$$+7''\cdot760 \sin(-\Omega + 184°\cdot42') + 0''\cdot256e \sin(-\Omega)$$
$$-4''\cdot382 \tan I \sin(\Omega + 169°\cdot51') + \text{short-period terms.}$$

With $e = \cdot055$, $\tan I = \cdot090$, this expression becomes

$$+8''\cdot108 \sin\Omega - 0''\cdot705 \cos\Omega + \text{short-period terms,}$$

which is therefore the equivalent in the longitude on the ecliptic of Hansen's terms with argument Ω.

I use $+7''\cdot112 \sin\Omega$, $+0''\cdot282 \sin(\Omega + 276°)$ for the ellipticity and planetary parts, respectively, of the terms with argument Ω. The latter of these has already been included in (*a*). Hence the correction to Hansen's tabular value still required is

$$-0''\cdot996 \sin\Omega + 0''\cdot705 \cos\Omega = +1''\cdot22 \sin(\Omega + 325°).$$

12. The time correction to the Hansen series from 1847 is only necessary in the years 1862–3. Dr. Cowell took his errors for 1847–61 from vol. l. of the *Monthly Notices*, and the correction to reduce to Leverrier's solar tables is there applied. From 1862 on, he states that he used the volumes of Greenwich observations. The solar tables of Leverrier were adopted in the *Nautical Almanac* in 1864, and these tables differ from those of Newcomb, as far as the mean motion of the Sun is concerned, by an amount which is insensible in the motion of the Moon. The change in the observed longitude of the Moon due to this cause is $0''\cdot30$ for 1862–3.

With respect to the correction for equinox, Mr. Hedrick writes:—

"The introductions to the several volumes of the *Greenwich Observations* from 1847 to 1902 state what Star Catalogue was used in the reduction of each year's observations. The introduction to the *Greenwich Catalogue*, 1845, states that the correction necessary to reduce all observations in 1847 to the equinox of that catalogue is $+0^s\cdot050$, and the introduction to the observations for 1848 states the corresponding correction for 1848 to be $+0^s\cdot062$. The necessary constant correction to reduce the several Greenwich Catalogues to Newcomb's equinox is given in *Astronomical Papers of the American Ephemeris*, vol. viii. p. 160; for the years 1899

to 1902 this correction is given in the *Greenwich Catalogue* for 1900 as $+0^{s}\cdot054$.

"The following table gives for each year, 1847 to 1902, the correction necessary to reduce the series to Newcomb's equinox—

Year.			Newc.-Gch.	Corr. to Obs. Place.
	s		s	″
1847	+0·050 to reduce to Gch.	'45	+0·074	+1·11
1848	+0·062 ,, ,,	'45	+0·086	+1·29
1849 to 1855	depends on ,,	'45	+0·024	+0·36
1856 ,, 1861	,, ,,	'50	+0·008	+0·12
1862 ,, 1869	,, ,,	'60	+0·024	+0·36
1870 ,; 1877	,, ,,	'64	+0·034	+0·51
1878 ,, 1888	,, ,,	'72	+0·029	+0·44
1889 ,, 1892	,, ,,	'80	+0·037	+0·56
1893 ,, 1898	,, ,,	5*yr* '90	+0·049	+0·74
1899 ,, 1902	,, ,,	10*yr* '90	+0·054	+0·81

"After 1902 Newcomb's Catalogue was used in the reduction of observations."

All these corrections are interpolated and inserted in col. (*c*) of Table II.

13. *The Long-Period Corrections* (*d*).—The correction on account of the great Venus term is

$$-15''\cdot34\sin(132^{\circ}t_c+214^{\circ}\cdot1)+14''\cdot27\sin(133^{\circ}t_c+212^{\circ}\cdot4),$$

the former being Hansen's value with changed sign, the latter is my value. Here t_c is the number of centuries from 1800.

The correction on account of the great empirical term is

$$-15''\cdot49\sin(132^{\circ}t_c+93^{\circ}\cdot9)+12''\cdot95\sin(131^{\circ}t_c+100^{\circ}\cdot6),$$

the former being Newcomb's value, now used in the *Nautical Almanac*, with changed sign; the latter is the value he obtained in his last investigation.

These two corrections are combined in the form,

$$+1''\cdot90\sin(132^{\circ}t_c+249^{\circ})+0''\cdot393t_c\sin(132^{\circ}t_c+335^{\circ}),$$

with sufficient accuracy for the observations from $t_c=-0\cdot5$ to $t_c=+1\cdot1$.

The planetary terms of very long period also included are:

$$+0''\cdot84\sin\{20^{\circ}\cdot2(t_c-0\cdot5)+41^{\circ}\cdot1\}$$
$$+0''\cdot04\sin\{4\mathrm{D}-3l+25\mathrm{M}-23\mathrm{T}+67^{\circ}\}$$
$$+0''\cdot31\sin\{l+3\mathrm{T}-10\mathrm{V}-2^{\circ}\cdot6(t_c-0\cdot5)+33^{\circ}\}.$$

On page 211 of his last investigation Newcomb, from a discussion of eclipses and occultations, obtains a correction to the

Hansen-Newcomb mean longitude, now used in the *Nautical Almanac*, of

$$0''\cdot83 + 2''\cdot60t_c - 0''\cdot47t_c^2,$$

where t_c is the number of centuries from 1800. For reasons stated below, I use this correction to the mean longitude. In forming the expression, however, Newcomb omits the three planetary terms of very long period just given, and for the coefficient of the great Venus term he uses $14''\cdot55$ instead of $14''\cdot27$, the value adopted in my work. My object being to represent the modern observations only, Newcomb's mean longitude must be changed so as to account for these four planetary terms. They are of such long period that with sufficient accuracy from 1750 to 1910, they may be represented by an expression of the form $a + bt + ct^2$. I tabulated them and analysed for such an expression, finding

$$+0''\cdot83 + 0''\cdot67t_c - 0''\cdot45t_c^2.$$

The maximum difference between this expression and the four terms is $0''\cdot06$ within the range.

I neglect this and the insensible difference $0''.02t_c^2$ and add to all the Greenwich observations the term

$$+1''\cdot93t_c.$$

The several corrections here noted are tabulated in col. (*d*) of Tables I, II.

14. *Corrections to Newcomb's Occultations.*—On page 30 of his memoir Newcomb gives as Hansen's tabular value for the terms with argument ☊,

$$\delta l = +7''\cdot594 \sin ☊ - 0''\cdot636 \cos ☊.$$

This is approximately the value for the longitude in the orbit.* He has compared it with Hill's value (which, however, refers to the longitude on the ecliptic), and has added the correction,

$$\delta l = +0''\cdot107 \sin ☊ + 0''\cdot363 \cos ☊.$$

Now the correct value of Hansen's terms in the ecliptic longitude is (No. 11 above)

$$+8''\cdot108 \sin ☊ - 0''\cdot705 \cos ☊.$$

Hence the value actually used by Newcomb is

$$+8''\cdot215 \sin ☊ - 0''\cdot342 \cos ☊$$

in the ecliptic longitude.

Newcomb states (p. 48) that the corrections of shorter period, amongst which these terms are included, can be applied indifferently to the longitude in the orbit or to the longitude on the ecliptic.

* A small error has been made in the reduction. Newcomb gives Hansen's terms in nz with argument $g \pm ☊$ correctly, but has added $0''\cdot140$ instead of $0''\cdot014$ as the portion of the coefficient of sin ☊, which they produce in the longitude.

This is not true in the case of these terms. As he has used Hansen's tables and has applied his corrections afterwards, I infer that the last expression really represents the terms used by Newcomb in the reduction of his occultations.

The value which I have adopted is

$$+7''\cdot112 \sin \Omega + 0''\cdot282 \sin (\Omega + 276^\circ)$$
$$= +7''\cdot140 \sin \Omega - 0''\cdot282 \cos \Omega .$$

Hence the addition to the tabular ecliptic longitude which must be applied to Newcomb's results in order to make them comparable with mine is

$$-1''\cdot075 \sin \Omega + 0''\cdot060 \cos \Omega .$$

This is the only correction applied. The three planetary terms of very long period and the change in the coefficient of the great Venus term are absorbed in the change of the mean longitude, as explained in No. 13 above.

15. Up to 1848·5 Newcomb gives mean results extending over a variety of intervals. I have not been able to discover the exact extent of these intervals from 1750 to this date, since no indication of them is given in Newcomb's paper. Dr. F. E. Ross has, however, kindly indicated their probable extent, and the brackets in the last column of Table I. show them. The epoch of each group is given by Newcomb: it may also be obtained from the diagram.

After 1848 Newcomb in almost all cases gives annual means. I have been unwilling to interpolate these so as to make them correspond to the periods P of the table, on account of the smoothing effect which would be produced. Difficulties of printing required them to be placed in a separate table (Table III.).

In Tables I, II, the first column gives the number of each period. In all cases the numbers in the body of the table are those appropriate to the *middle* of each numbered period. The middle of period 1 is the date 1751·29, and any other date can be easily obtained from the relation, $9P = 10\cdot20$ years.

Columns (a), (b), (d), and (a), (c), (d) contain the corrections noted above in units of $0''\cdot01$; the sum of these is given in the following columns under the heading "Cor." Columns C. contain the errors to $0''\cdot1$ of the Moon's place, tabular–observed, as given by Dr. Cowell in *M.N.*, vol. lxv. pp. 38, 39. Columns "Mer." are equal to Cor. + C., and therefore contain the final residuals derived from the Greenwich meridian observations. Column "Occ." in Table I. contains the results derived by Newcomb from the occultations and given on p. 211 of his paper after correction for the node term noted in No. 14 above.

Table III. gives Newcomb's results, with the same correction, for the years 1848–1908.

The final meridian and occultation results are shown in the diagram. In order to distinguish between them, the meridian

residuals for each period are joined by straight lines; the occultation residuals are shown by heavy dots. There is, fortunately, no confusion in the overlapping Airy and Hansen periods, Nos. 86–89. The periods are set down on the line of zero ordinates; the years are shown below.

TABLE I.

P.	(*a*)	(*b*)	(*d*)	Cor.	C.	Mer.	Occ.
1	− 11	− 57	− 38	−1″06	+3″6	+2″5	
2	+ 7	− 77	− 40	−1·10	+0·5	−0·6	
3	+ 19	−146	− 43	−1·70	+1·2	−0·5	
4	+ 22	−128	− 46	−1·52	+1·4	−0·1	+0″1
5	+ 16	− 63	− 49	−0·96	+0·1	−0·9	
6	+ 3	− 55	− 52	−1·04	+1·0	0·0	
7	− 11	+ 3	− 54	−0·62	+1·2	+0·6	
8	− 23	+ 49	− 57	−0·31	+2·0	+1·7	
9	− 31	+ 57	− 60	−0·34	+1·6	+1·3	
10	− 36	+ 46	− 63	−0·53	+2·3	+1·8	
11	− 41	− 36	− 66	−1·43	+2·1	+0·7	
12	− 46	− 44	− 68	−1·58	+3·4	+1·8	
13	− 47	− 34	− 70	−1·51	+0·4	−1·1	
14	− 40	− 84	− 73	−1·97	+2·5	+0·5	
15	− 22	− 88	− 75	−1·85	+2·5	+0·6	
16	+ 7	− 79	− 77	−1·49	+1·8	+0·3	
17	+ 42	− 89	− 79	−1·26	+0·3	−1·0	
18	+ 76	−101	− 81	−1·06	−0·8	−1·9	
19	+103	−161	− 84	−1·42	+1·2	−0·2	−1·2
20	+119	−119	− 86	−0·86	+1·7	+0·8	
21	+121	− 52	− 89	−0·20	0·0	−0·2	
22	+112	− 57	− 90	−0·35	−0·7	−1·0	
23	+ 95	+ 14	− 92	+0·17	−1·7	−1·5	
24	+ 76	+ 73	− 94	+0·55	−0·6	0·0	
25	+ 58	+ 88	− 96	+0·50	−0·9	−0·4	
26	+ 42	+ 43	− 98	−0·13	−2·7	−2·8	
27	+ 26	− 40	− 99	−1·13	−0·1	−1·2	
28	+ 12	− 46	−101	−1·35	+1·3	0·0	
29	0	− 72	−103	−1·75	−0·3	−2·0	
30	− 7	−147	−104	−2·58	+0·5	−2·1	
31	− 6	−106	−106	−2·18	+0·9	−1·3	−1·7
32	+ 2	− 80	−106	−1·84	+0·6	−1·2	
33	+ 14	− 94	−107	−1·87	+1·0	−0·9	
34	+ 28	−113	−108	−1·93	−1·1	−3·0	

TABLE I.—*continued.*

P.	(*a*)	(*b*)	(*d*)	Cor.	C.	Mer.	Occ.
				"	"	"	
35	+ 41	−136	−108	−2·03	−0·5	−2·5	
36	+ 49	− 74	−109	−1·34	−1·5	−2·8	
37	+ 51	− 43	−108	−1·00	−1·4	−2·4	
38	+ 44	− 32	−109	−0·97	−1·7	−2·7	−0·3
39	+ 29	+ 52	−109	−0·28	−2·3	−2·6	
40	+ 8	+ 68	−109	−0·33	−1·1	−1·4	
41	− 13	+ 25	−110	−0·98	+2·4	+1·4	
42	− 28	− 29	−109	−1·66	+2·1	+0·4	
43	− 32	− 95	−109	−2·36	+1·7	−0·7	
44	− 26	− 72	−108	−2·06	+2·4	+0·3	
45	− 11	− 73	−108	−1·92	+2·0	+0·1	
46	+ 7	− 88	−107	−1·88	+2·7	+0·8	+0·6
47	+ 26	− 40	−105	−1·19	+1·9	+0·7	
48	+ 44	− 54	−105	−1·15	+2·8	+1·6	
49	+ 61	−101	−104	−1·44	+3·4	+2·0	
50	+ 77	−144	−103	−1·70	+2·6	+0·9	
51	+ 94	−162	−102	−1·70	+2·8	+1·1	
52	+107	− 67	− 99	−0·59	+1·0	+0·4	
53	+116	− 26	− 98	−0·08	+0·6	+0·5	+0·3
54	+116	− 10	− 96	+0·10	+2·1	+2·2	
55	+106	+ 59	− 94	+0·71	+1·6	+2·3	
56	+ 85	+ 30	− 92	+0·23	+2·2	+2·4	+0·6
57	+ 58	− 41	− 89	−0·72	+4·7	+4·0	
58	+ 32	− 98	− 86	−1·52	+3·8	+2·3	
59	+ 12	−104	− 84	−1·76	+4·6	+2·8	
60	+ 2	− 39	− 82	−1·19	+3·8	+2·6	
61	+ 3	− 43	− 79	−1·19	+5·2	+4·0	+0·2
62	+ 9	− 54	− 75	−1·20	+4·5	+3·3	
63	+ 15	− 7	− 72	−0·64	+3·3	+2·7	
64	+ 18	− 68	− 69	−1·19	+3·6	+2·4	+0·1
65	+ 17	−136	− 65	−1·84	+3·4	+1·6	
66	+ 15	−142	− 61	−1·88	+3·1	+1·2	
67	+ 14	−104	− 56	−1·46	+2·9	+1·4	
68	+ 16	− 32	− 52	−0·68	+2·0	+1·3	
69	+ 18	− 34	− 48	−0·64	+2·5	+1·9	+1·6
70	+ 17	− 28	− 43	−0·54	+1·2	+0·7	
71	+ 11	+ 5	− 39	−0·23	+1·7	+1·5	

TABLE I.—*continued.*

P.	(*a*)	(*b*)	(*d*)	Cor.	C.	Mer.	Occ.
72	0	− 76	− 33	−1″·09	+3″·3	+2″·2	
73	− 16	−120	− 29	−1·65	+4·5	+2·8	
74	− 32	−104	− 24	−1·60	+3·8	+2·2	+0″·8
75	− 43	− 63	− 19	−1·25	+3·4	+2·2	
76	− 45	+ 29	− 14	−0·30	+2·6	+2·3	
77	− 38	+ 36	− 8	−0·10	+2·4	+2·3	
78	− 23	+ 42	− 3	+0·16	+2·2	+2·4	
79	− 5	+ 43	+ 2	+0·40	+1·1	+1·5	+0·6
80	+ 12	− 50	+ 7	−0·31	+2·2	+1·9	
81	+ 25	−103	+ 13	−0·65	+1·8	+1·2	
82	+ 35	−117	+ 19	−0·63	+1·2	+0·6	−0·2
83	+ 43	− 94	+ 25	−0·26	+0·6	+0·3	
84	+ 50	− 40	+ 31	+0·41	−1·0	−0·6	
85	+ 54	− 51	+ 37	+0·40	−0·1	+0·3	−0·5
86	+ 54	− 27	+ 43	+0·70	−0·6	+0·1	
87	+ 50	+ 12	+ 51	+1·13	−2·1	−1·0	−2·0
88	+ 41	− 42	+ 57	+0·56	−1·6	−1·0	−0·6
89	+ 30	− 61	+ 64	+0·33	−0·7	−0·4	−1·8

TABLE II.

P.	(*a*)	(*c*)	(*d*)	Cor.	C.	Mer.
86	+54	−243	+ 43	−1″·46	+0″·6	−0″·9
87	+50	−263	+ 51	−1·62	−0·4	−2·0
88	+41	−110	+ 57	−0·12	−1·2	−1·3
89	+30	−121	+ 64	−0·27	−1·8	−2·1
90	+18	−129	+ 71	−0·40	−2·3	−2·7
91	+ 6	− 60	+ 78	+0·24	−2·6	−2·4
92	− 6	− 56	+ 85	+0·23	−2·9	−2·7
93	−16	−120	+ 92	−0·44	−2·6	−3·0
94	−20	− 51	+ 99	+0·28	−3·3	−3·0
95	−18	+ 62	+106	+1·50	−4·3	−2·8
96	−12	+ 93	+114	+1·95	−4·7	−2·8
97	− 3	+114	+122	+2·33	−5·3	−3·0
98	+ 1	+186	+129	+3·16	−6·3	−3·1
99	− 1	+109	+136	+2·44	−5·8	−3·4
100	− 9	− 13	+143	+1·21	−5·0	−3·8
101	−21	− 79	+150	+0·50	−3·7	−3·2
102	−32	− 83	+159	+0·44	−3·0	−2·6

TABLE II.—*continued.*

P.	(a)	(c)	(d)	Cor.	C.	Mer.
103	−39	−145	+166	−0″·18	−1″·8	−2″·0
104	−42	−173	+174	−0·41	−1·0	−1·4
105	−41	− 94	+181	+0·46	−1·7	−1·2
106	−40	− 88	+189	+0·61	−1·4	−0·8
107	−42	−159	+198	−0·03	+0·1	+0·1
108	−45	−145	+205	+0·15	+0·2	+0·4
109	−49	− 77	+213	+0·87	+0·3	+1·2
110	−49	− 70	+221	+1·02	+0·2	+1·2
111	−43	− 37	+229	+1·49	+0·2	+1·7
112	−28	− 12	+238	+1·98	+0·3	+2·3
113	−10	+139	+246	+3·75	−1·0	+2·8
114	+ 5	+ 74	+254	+3·33	−1·3	+2·0
115	+13	+ 48	+262	+3·23	−0·8	+2·4
116	+14	+ 38	+270	+3·22	−1·0	+2·2
117	+ 9	− 56	+278	+2·31	+0·1	+2·4
118	+ 5	−120	+286	+1·71	+0·5	+2·2
119	+ 6	− 75	+294	+2·25	+0·2	+2·4
120	+10	− 72	+302	+2·40	+0·7	+3·1
121	+16	−140	+309	+1·85	+0·9	+2·8
122	+17	−146	+318	+1·89	+1·0	+2·9
123	+14	−130	+326	+2·10	+0·4	+2·5
124	+ 6	−179	+333	+1·60	+0·7	+2·3
125	− 4	−184	+341	+1·53	+1·4	+2·9
126	−15	− 77	+348	+2·56	+1·0	+3·6
127	−22	− 3	+356	+3·31	0·0	+3·3
128	−26	+ 10	+363	+3·47	−0·4	+3·1
129	−28	+ 70	+371	+4·13	−1·0	+3·1
130	−29	+117	+378	+4·66	−1·8	+2·9
131	−32	+ 36	+386	+3·90	−1·8	+2·1
132	−37	− 49	+393	+3·07	−1·5	+1·6
133	−43	− 50	+400	+3·07	−1·9	+1·2
134	−46	− 79	+407	+2·82	−2·0	+0·8
135	−46	− 99	+414	+2·69	−2·3	+0·4
136	−40	− 69	+420	+3·11	−3·5	−0·4
137	−32	− 41	+428	+3·55	−5·0	−1·4
138	−26	− 99	+434	+3·09	−5·4	−2·3
139	−22	−145	+441	+2·74	−6·0	−3·3
140	−22	−102	+447	+3·23	−6·5	−3·3
141	−23	− 67	+453	+3·63	−7·0	−3·4

TABLE III.

Newcomb's results corrected. Tabular–observed errors.

Year.	1840+	1850+	1860+	1870+	1880+	1890+	1900+
0·5		−1″·8	−3″·0	−1″·7	+2″·5	+2″·3	+1″·0
1·5		−1·8	−2·2	+0·2	+2·6	+2·2	+0·9
2·5		−2·0	−2·8	+0·8	+2·3	+1·9	+0·2
3·5		−1·7	−2·0	+1·0	+2·8	+2·3	−0·4
4·5		−2·3	−2·2	+1·7	+2·4	+3·0	−1·3
5·5		−2·7	−2·2	+2·1	+2·4	+2·5	−2·0
6·5		−2·2	−2·7	+2·3	+2·4	+1·8	−2·1
7·5		−2·1	−1·4	+2·5*	+1·9	+1·9	−3·0
8·5	−2″·0	−3·1	−1·6		+2·6	+2·5	−3·2†
9·5	−0·6	−3·2	−2·5	+1·5	+2·4	+2·5	

Change of the Secular Acceleration and the Empirical Term.

16. The original purpose of this work was a determination of the mean longitude which should best satisfy the modern observations. Owing to the presence of unknown forces producing periodic changes whose amounts are not known, the solution is not unique. For purposes of comparison I adopted Newcomb's last values for the mean longitude and the great empirical term. In finding them he considered the secular acceleration as one of the unknowns and found it from a combination of modern observations and ancient eclipses.

Suppose that we consider modern observations alone, and examine the errors which will be produced by changes in the mean longitude and the great empirical term, such that the observations are still satisfied within the limits of error. This last phrase has really no definite meaning in this connection. But we can settle beforehand the nature of the errors which we are about to introduce. Since there are periodic errors still existing, and since the object is to avoid new secular errors within the range of the observations in making any changes, it seems best to so determine these changes that, if possible, only periodic errors within the range are introduced.

The question is: Can we so determine the great empirical term with a change in the secular acceleration and, if necessary, in the epoch and mean motion that the errors produced are approximately periodic within reasonable limits from 1750 to 1910 and within the probable errors from 1621 to 1750?

The secular acceleration used by Newcomb is $1''\cdot88t_c^2$ greater than my theoretical sidereal value $6''\cdot08t_c^2$. Let us take this as the change to be made. We have five constants at our disposal, the

* The epoch is 1878·0. † The epoch is 1908·3.

epoch of mean longitude, the mean motion, and the three constants of the empirical term. The conditions laid down will be approximately satisfied if we make the excess of the secular acceleration plus Newcomb's empirical term equal to a new empirical term plus changes in the mean longitude, at five specified dates.

17. Let $$A = ct^2 + a \sin mt + b \cos mt$$

be the excess of the secular acceleration plus Newcomb's empirical term used above, and

$$B = u + vt + x \sin zt + y \cos zt$$

additions to the mean longitude plus a new empirical term. Let $B = A$ when $t = \pm 2, \pm 1, 0$, the dates being taken as equidistant for ease in computation.

Solving the five equations thus obtained, I find

$$\cos z = \frac{2c + b(\cos 2m - 2\cos m + 1)}{2c + 2b(\cos m - 1)}, \quad y = \frac{c + b(\cos m - 1)}{\cos z - 1},$$

$$x = a\frac{\sin 2m - 2\sin m}{\sin 2z - 2\sin z}, \quad v = a \sin m - x \sin z, \quad u = b - y.$$

The epoch $P = 67{\cdot}5$ is near the middle of the Greenwich series; take $t = 0$ there. Choose the unit of time equal to $45P = 51$ years, as this makes a fair distribution over the series. Then, changing from t_c to the new unit of time, I find

$$c = +0''{\cdot}49, \quad a = -9''{\cdot}11, \quad b = +9''{\cdot}20, \quad m = 66°{\cdot}8.$$

The solution of the equations produces

$$B = +1''{\cdot}56 - 0''{\cdot}73t + 11''{\cdot}15 \sin(70°{\cdot}4t + 136°{\cdot}7).$$

The period of the new term is 261 years as against the 273 years of Newcomb's term, and the coefficient is nearly 2″ less.

Table IV. shows the errors produced. It will be noticed that between 1710 and 1930 there is no error so great as 0″·1, and that, for the century before, the errors are all less than 0″·5. These are much smaller than anything systematic (periodic) which could be deduced from the observations. At least 80 more years will be required to distinguish between the terms A, B.

TABLE IV.—*Change of the Empirical Term.*

Date.	t.	ct^2.	$a \sin mt.$ $+b \cos mt.$	A.	$u+vt.$	$x \cos zt.$ $+y \sin zt.$	B.	A−B.
1622·7	−4·0	+7·82	− 9·54	− 1·72	+4·48	− 6·41	− 1·93	+0·21
1632·9	−3·8	+7·06	−11·31	− 4·25	+4·33	− 8·42	− 4·09	−0·16
1643·1	−3·6	+6·34	−12·46	− 6·12	+4·19	− 9·95	− 5·76	−0·36
1653·3	−3·4	+5·65	−12·94	− 7·29	+4·04	−10·88	− 6·84	−0·45
1663·5	−3·2	+5·01	−12·72	− 7·71	+3·90	−11·15	− 7·25	−0·46
1673·7	−3·0	+4·40	−11·80	− 7·40	+3·75	−10·74	− 6·99	−0·41
1683·9	−2·8	+3·83	−10·26	− 6·43	+3·60	− 9·70	− 6·10	−0·33
1694·1	−2·6	+3·31	− 8·15	− 4·84	+3·46	− 8·06	− 4·60	−0·24
1704·3	−2·4	+2·82	− 5·60	− 2·78	+3·31	− 5·94	− 2·63	−0·15
1714·5	−2·2	+2·37	− 2·75	− 0·38	+3·16	− 3·48	− 0·32	−0·06
1724·7	−2·0	+1·96	+ 0·25	+ 2·21	+3·01	− 0·80	+ 2·21	0·00
1734·9	−1·8	+1·58	+ 3·24	+ 4·82	+2·87	+ 1·94	+ 4·81	+0·01
1745·1	−1·6	+1·25	+ 6·05	+ 7·30	+2·73	+ 4·55	+ 7·28	+0·02
1755·3	−1·4	+0·96	+ 8·52	+ 9·48	+2·58	+ 6·90	+ 9·48	0·00
1765·5	−1·2	+0·70	+10·54	+11·24	+2·43	+ 8·82	+11·25	−0·01
1775·7	−1·0	+0·49	+11·99	+12·48	+2·28	+10·21	+12·49	−0·01
1785·9	−0·8	+0·31	+12·79	+13·10	+2·14	+10·99	+13·13	−0·03
1796·1	−0·6	+0·18	+12·91	+13·09	+1·99	+11·12	+13·11	−0·02
1806·3	−0·4	+0·08	+12·32	+12·40	+1·85	+10·57	+12·42	−0·02
1816·5	−0·2	+0·02	+11·06	+11·08	+1·70	+ 9·38	+11·08	0·00
1826·7	0·0	+0·00	+ 9·20	+ 9·20	+1·55	+ 7·65	+ 9·20	0·00
1836·9	+0·2	+0·02	+ 6·85	+ 6·87	+1·41	+ 5·44	+ 6·85	+0·02
1847·1	+0·4	+0·08	+ 4·13	+ 4·21	+1·26	+ 2·90	+ 4·16	+0·05
1857·3	+0·6	+0·18	+ 1·18	+ 1·36	+1·12	+ ·19	+ 1·31	+0·05
1867·5	+0·8	+0·31	− 1·84	− 1·53	+0·97	− 2·53	− 1·56	+0·03
1877·7	+1·0	+0·49	− 4·75	− 4·26	+0·82	− 5·08	− 4·26	0·00
1887·9	+1·2	+0·70	− 7·40	− 6·70	+0·68	− 7·34	− 6·66	−0·04
1898·1	+1·4	+0·96	− 9·66	− 8·70	+0·53	− 9·17	− 8·64	−0·06
1908·3	+1·6	+1·25	−11·39	−10·14	+0·39	−10·44	−10·05	−0·09
1918·5	+1·8	+1·58	−12·51	−10·93	+0·24	−11·08	−10·84	−0·09
1928·7	+2·0	+1·96	−12·94	−10·98	+0·09	−11·06	−10·97	−0·01
1938·9	+2·2	+2·37	−12·68	−10·31	−0·05	−10·37	−10·42	+0·11
1949·1	+2·4	+2·82	−11·73	− 8·91	−0·19	− 9·06	− 9·25	+0·34
1959·3	+2·6	+3·31	−10·14	− 6·83	−0·34	− 7·20	− 7·54	+0·71
1969·5	+2·8	+3·83	− 8·02	− 4·19	−0·48	− 4·91	− 5·39	+1·20
1979·7	+3·0	+4·40	− 5·45	− 1·05	−0·63	− 2·34	− 2·97	+1·92

18. In an earlier part of this investigation, before all the new corrections had been applied, I deduced an excess $5''\cdot41t_c^2$ of the secular acceleration above the theoretical value from the Greenwich observations alone and solved the equations giving the changes of the mean longitude and the empirical term which were necessary to

account for the excess. Although the results have no immediate application, it seems worth while to show the extent of the changes and errors produced by so large a change in the secular acceleration.

With $c = 1''\cdot41t^2$ and Newcomb's last empirical term, I found

$$B = +4''\cdot06 - 2''\cdot11t + 8''\cdot19 \sin(79^\circ\cdot08t + 141^\circ\cdot13).$$

The period is 232 years, and the coefficient is less than two-thirds that of Newcomb's term. The errors A–B are shown in the following table. They are less than $0''\cdot15$ for the Greenwich series, and less than $1''\cdot6$ back to 1620.

Date.	+0	+51	+102	+153	+204	+255	+306	+357
	"	"	"	"	"	"	"	"
1622·7	+1·16	−1·39	0·00	0·00	0·00	0·00	+0·00	+6·90
1632·9	−0·19	−1·13	+0·11	−0·07	+0·06	−0·11	+0·49	
1643·1	−1·03	−0·80	+0·14	−0·09	+0·11	−0·23	+1·35	
1653·3	−1·43	−0·47	+0·12	−0·07	+0·12	−0·29	+2·67	
1663·5	−1·52	−0·20	+0·07	−0·04	+0·07	−0·25	+4·49	

19. There are certain limitations to this method aside from the errors which may be produced by the change from the terms A to the terms B. If we retain *equidistant* dates for the equality of the two expressions, the epoch of the central date and the common interval between the dates are not arbitrary without limitation. The inequality $-1 < \cos z < +1$ is easily shown to require the satisfaction of the inequality,

$$\sin a \sin {}^2m < 2c/C,$$

where $\tan a = b/a$, so that a is the phase of the given empirical term at the central date, and C is the absolute value of its coefficient. Further, to avoid very large errors, B–A, values of a equal to or near a multiple of 90° must be avoided. The actual choice of the central date and of the interval of 51 years in the two cases worked out above was made without a knowledge of these conditions: the errors B–A, which are found within the range of 160 years, are probably near their minima for all values of a, when the values of c and the given empirical term are fixed.

20. Dr. Cowell has deduced an empirical term of 320 to 350 years' period, according to the value of the secular acceleration adopted, from the Greenwich observations. I have made analyses of the same observations, but with the new corrections applied, and have in all of them obtained a positive correction to the Hansen-Newcomb secular acceleration; this requires that the period and coefficient of Newcomb's empirical term must be lessened when we reduce the secular acceleration to its theoretical value. Probably this difference between our results is mainly due to the new corrections which I have applied, but the mode of analysis may partly account for it. By taking means for 17 consecutive periods Dr. Cowell obtains 31 numbers, from which he extracts ten unknowns, which correspond to the ten coefficients of a power series in t as far as t^9. From these he extracts three periodic terms and a constant addition to the mean motion. I analysed for terms of

the form $a + bt + ct^2$, and, after subtracting them from the observations, found the 60- to 70-year term by the periodogram method. The observations, freed from this term, were again analysed for terms of the form $a + bt + ct^2$. The terms of shorter period were obtained by the periodogram method, the whole process being repeated after the extraction of the terms in order to avoid the influence of one term on another.

21. The existence of these terms of shorter period makes the determination of the secular acceleration and the great empirical term from the Greenwich series a doubtful one. And the observations before 1840 are not sufficiently accurate for the purpose: by the omission of the values at two periods, which seemed to be seriously in error, a change of about 1″ in the value deduced for the secular acceleration by the method I used was found. I conclude that we must either adopt the theoretical value of the secular acceleration or deduce an observed value from a combination of the ancient eclipses and modern observations, and that for the great empirical term we must make use of the occultations before 1750 in combination with the meridian observations and occultations after that date.

Comparison of the Meridian Observations and Occultations.

22. The differences are most easily seen on the diagram. They are comparatively large near 1793 and 1820, the latter being about 2″. In spite of a long search I have not been able definitely to locate the cause of these differences, but the possible sources of systematic error, more particularly in the occultations, will be briefly shown.

23. In the early occultations there is a great inequality in the distribution with respect to right ascension. I made a count of the number of occultations in different zones, which is shown in the following table, where, however, the limits of R.A. are only approximate:—

R.A. Date.	350°–50°	50°–90°	90°–140°	140°–170°	170°–220°	220°–250°	250°–300°	300°–350°
1753–79	4	25	10	3	5	5	0	7
1783–01	11	24	8	9	11	8	4	3
1801–20	14	49	10	9	0	11	6	17
1821–22	7	62	3	7	0	0	0	1
1823–28	7	9	19	5	6	3	4	6
1829–31	12	65	8	6	1	9	2	1
1832–35	2	5	0	5	4	5	11	9
1836–38	13	6	4	3	2	3	5	5
1839–40	6	27	3	0	4	12	7	17
1841–44	7	24	11	6	0	2	3	3
1845–47	8	11	8	11	0	4	8	5
1848–49	17	31	4	5	5	7	3	5
1850	7	22	7	10	0	4	5	5

Large differences due to R.A. errors can hardly be caused by systematic errors in the star places, for Newcomb reduces all of them to his catalogue, and there can be little doubt that it is nearly free from such errors. The meridian observations are necessarily well distributed in R.A., so that such errors should be nearly eliminated in taking means over 414 days. But if tabular errors of periods nearly equal to a month due to unknown forces exist (we know similar errors of long period exist), they would produce apparent errors of long period in the occultations. It may be noticed that the greatest errors occur in 1821–2, where the inequality in the distribution is greatest, but the correlation cannot be further developed until the supposed short-period terms can be obtained in other ways.

24. Another source of error may arise from Newcomb's method of combining the occultations. In certain groups he uses many observations on two or three nights, and very few for the rest of the period. A separate equation of condition is used for each occultation; a short-period term may again cause an apparent systematic error in the group. At my request, Mr. Hedrick solved Newcomb's groups for the epochs 1821·0, 1822·5, combining all the observations for each night into a single equation of condition, giving it weight two when the number of observations exceeded three. But when, as a test, he followed Newcomb's instructions so as to reproduce the printed values, he obtained quite different results. And we were not able to find out exactly how Newcomb did deduce his group values.

When group errors of the order of 1″ are under consideration, the particular system of weights to be adopted is also important. Different methods of combination and weighting, more particularly with respect to the immersions and emersions, will modify the results of some of Newcomb's groups very materially. And as different workers will use different methods and weights, one is forced to the conclusion that one or two errors of 2″ in Newcomb's groups from 1750 to 1850 are not improbable.

25. When there are several occultations observed at two or more places on the same night, a systematic difference is sometimes noted. I made a count of the number of occultations used by Newcomb for groups of years and places, which is shown in the following table:—

Place. Dates.	Greenwich.	Camb. Eng.	Havana.	Paris.	Dorpat.	Dorchester.	Cracow.	Berlin.	Cape.
1753–93	102	1							
1794–01	38	21							
1802–09	18		11						
1810–14	12	11	12	27	13				
1816–24		14		32	56				
1825–30	16	12		13	5	18	20	14	
1831–35	19	3				17	29	12	11
1836–38							16	16	12

This tells little more, however, than the fact that the agreement between the occultations and the meridian observations is best in those periods where the Greenwich observations are relatively most numerous, and worst where the Dorpat observations have the chief influence on the results.

26. A fourth source of error, occurring in both sets, arises from the limb used. For the meridian observations I had a count made of the number of first limb and second limb observations in each period from 1750 to 1850; the difference was never great enough to cause an error of more than 0″·2 or 0″·3 in any period as compared with an equal number of observations of each limb. The number of observations per period in this interval ranges from 47 to 178. The error is likely to be more serious with the occultations, owing to the conditions of observation, and Newcomb, recognising this fact, has solved separately for immersions and emersions, and has devoted some space to their discussion. The difference in one group is upwards of 3″. He has made certain corrections on this account, but these are necessarily quite doubtful: I am inclined to believe that the main differences between the results deduced from the occultation and meridian observations must be set down to this cause.

27. While no definite result has been deduced from the mention of these sources of error, it seemed worth while to state a part of what has been done towards their investigation and to give the statistics. The general agreement is good, but we cannot find the periodic changes of shorter period with much certainty until the differences which occur during the first half of the nineteenth century are explained.

28. There is one peculiarity in these minor residuals which is particularly striking in the meridian observations since 1850. A reference to the diagram shows that they may be represented quite closely by four straight lines which change their inclinations near periods 100 (1863), 112 (1877), 130 (1897). The occultations do not show the peculiarity in so marked a manner, but the oscillations of the latter from year to year seem to indicate a lower degree of accuracy. Before 1850 the degree of accuracy of the meridian observations is apparently not high enough to show this feature well, but straight lines may still represent the residuals, with changes in inclination about periods 35 (1790), 60 (1818). The most marked of all these changes is at period 100. The mean motion of the Moon suddenly changed there by nearly 1″ per annum, this change persisting for twelve or thirteen periods until the smaller changes at period 112 and later at period 130 took place. There is also some indication of a new change at period 139 (1908).

These apparently sudden changes may be the combined effect of several periodic terms, but the possibility of their real existence should not be excluded in an attempt to account for the differences between theory and observation. I shall not now discuss hypotheses which might be brought forward to explain them.

29. Besides the investigations into the equinox and time changes noted above, Mr. H. B. Hedrick has performed the greater part of the tabulations and calculations needed in this work. He has also assisted much in discussions on possible sources of errors in the observational material. The expense has been met out of the Lunar Tables Fund provided by Yale University.

Yale University:
1913 *June* 25.

On the Expression of Sun-spot Periodicity as a Fourier Sequence, and on the General Use of a Fourier Sequence in Similar Problems. (Papers of the I.U.S.R. Computing Bureau, No. X.) By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. In a former paper (*M.N.*, lxxiii. p. 549) I explained how I had been led by the work of Mr. Kimura to the calculation of the Fourier sequence of exact submultiples of 156 years, which is approximately the length of the period for which we have available Wolf's "Sun-spot numbers." The further study of these results has led me to the conclusion that this method of expression has advantages over the "periodogram" proposed by Professor Schuster. The advantages are in detail only, the method being in essence the same as that of the periodogram; but they are very definite advantages and have led to the clearing up of several difficulties which the use of the periodogram had raised. The method has not yet been tried on any other example, but its merits seem to be general enough, and a brief account of them is therefore adventured before leaving this problem for another.

2. The essential feature of the method is that we should divide the whole period over which the observations extend into exact submultiples, and calculate the sine and cosine of the harmonics for these submultiple periods and for these only. The length of the whole period of observation is of course continually changing, and we must make our choice at some arbitrary point. The choice is, however, not important, seeing that the addition or subtraction of a few observations, presumably consistent with the general mass, ought not to make any serious difference. We need not even adjust the ends of the series to fit as well as possible, as I thought when writing the former paper (*loc. cit.*, p. 550), and the only point to which it seems convenient to pay attention, if we use the strict Fourier method, is the choice of a whole number of observations with convenient factors. Thus in the present case $156\ (=12\times13)$ is preferable to 157 or 155.

3. The advantages of this process may be thus summarised:—

(*a*) It is *necessary*,
(*b*) It is *sufficient*,
(*c*) It is *convenient*,
(*d*) It is *instructive*,

and we will take these points in order.

(*a*) *The Fourier Sequence is necessary.*

4. Let us first compare the process with that of the periodogram as outlined by Professor Schuster. In discussing the closeness of the periods which should be examined separately, he writes [*Phil. Trans.*, 206, A (1906), p. 71]:—

> The different periods . . . should be chosen near together, but there is a limit beyond which it would be useless to go. This limit is reached when the values of A and B for two closely adjoining values n_1 and n_2 are no longer independent of each other. The theory of vibration shows that independence begins when there is an ultimate disagreement of phase amounting to about one quarter of a period.

5. Now, the characteristic of the Fourier sequence is that each term runs through exactly *one whole* period more than the last. Hence I advocate calculating about *one quarter* of the terms suggested in the above paragraph. All that Professor Schuster has therefore urged on behalf of the necessity of the periodogram applies with quadruple force to the Fourier sequence: and the matter might be left there.

6. But the necessity can be put in even simpler form. Since each term of the Fourier sequence is independent of every other, it cannot be inferred from any other. Hence we must *at least* calculate all these terms.

(*b*) *The Fourier Sequence is sufficient.*

7. It is well known that the Fourier sequence represents the observations which can be reproduced from the sequence when fully calculated. But the series is infinitely long, and therefore cannot be fully calculated. Nevertheless, it is tolerably obvious that, if the observations show anything like regularity, the terms of high order will be small, and to neglect them will enable us to reproduce the original observations *very nearly* by means of the terms retained. Thus if there be (in Sun-spots) a periodicity of a few weeks, we can disregard it in considering periods near 8–12 years. Hence we may say that we can *practically* reproduce the observations from the Fourier sequence, and there must be in the sequence everything that was in the observations. For instance, if we desire to know to what extent any periodicity intermediate between two of those directly tabulated is represented in the observations, we must be able to deduce this information from the sequence. But though this proves the point theoretically, it might turn out practically that it is much more troublesome to get this information from the sequence than from the observations themselves. Hence we must examine the nature of the deduction from the sequence.

8. Let us first inquire what happens when there is a single real periodicity. For simplicity, let us first suppose the number of observations very large, so that we may use integration instead of

summation. [The corresponding calculation for a finite series is rather more complex, and is given in § 48 in a separate note, in order not to interrupt the argument unduly.] Let us represent the whole length of the series by 2π, and the argument by t; and let us suppose that the real periodicity has coefficient unity and is represented by .

$$\sin(pt+\alpha),$$

where α is the phase at starting. By hypothesis p is not to be an integer: otherwise the term would run through its period an exact number of times in 2π—in other words, it would coincide with one of the terms of the Fourier sequence. To find the effect of this periodicity on any term q of the Fourier sequence we have to form the integrals

$$\pi S_q = \int_0^{2\pi} \sin(pt+\alpha)\sin qt\,dt,$$

and

$$\pi C_q = \int_0^{2\pi} \sin(pt+\alpha)\cos qt\,dt.$$

Let us take the first of them: we have

$$2\pi S_q = \int_0^{2\pi} \cos[(p-q)t+\alpha]dt - \int_0^{2\pi} \cos[(p+q)t+\alpha]dt.$$

Now when we come to integrate, we have

$$\sin[(p\pm q)2\pi+\alpha] = \sin(2\pi p+\alpha),$$

since q is an integer, and therefore $2\pi q$ may be omitted without disturbance. The two terms will thus only differ in the coefficients $1/(p-q)$ and $1/(p+q)$, and we find

$$\pi S_q = \frac{q}{p^2-q^2}\,.\,\{\sin(2\pi p+\alpha)-\sin\alpha\}$$

$$= \frac{2q}{p^2-q^2}\,.\,\sin\pi p\,.\,\cos(\pi p+\alpha).$$

Similarly

$$\pi C_q = \frac{2p}{p^2-q^2}\,.\,\sin\pi p\ \sin(\pi p+\alpha).$$

9. We can thus readily trace the effect of the periodicity on the successive terms of the Fourier sequence, for which q takes successive integral values. If p lies between q and $q+1$, there will clearly be a change of sign in both S_q and C_q as we pass from q to $q+1$, since p^2-q^2 will change sign once and once only, q remaining essentially positive, and the other factors being independent of q. This change of sign is here regarded as of great importance. It gives us a new criterion for detecting the presence of a striking periodicity from the *signs* of S and C—a criterion which is clearly lost when we form S^2+C^2 as in the periodogram. Further, it is submitted that this criterion is better able to indicate the precise value of the period in question (presuming it to be well defined)

than the periodogram method of a maximum of $S^2 + C^2$. Maxima are apt to be ill-defined, and even if regular may be flat. The epoch of a sinuous curve is best determined, not from its maxima and minima, but from the rising or falling portions: and the new criterion utilises a property of this kind. But to show this clearly we must first trace the changes of magnitude of S_q and C_q.

10. The magnitudes are large when $p^2 - q^2$ is small, so that we may write q for p in the numerator of C without serious error. If we adopt this approximation, we have

$$S_q/C_q = \cot(\pi p + \alpha),$$

which is independent of q; and it will thus suffice to consider either S_q or C_q or $R_q = (S_q^2 + C_q^2)^{\frac{1}{2}}$. Let us put $p = q + x$. Then $\sin \pi p = (-1)^q \sin \pi x$,

and
$$\pi R_q = (-1)^q \cdot \frac{2q}{2q+x} \cdot \frac{\sin \pi x}{x}.$$

The first factor settles the sign merely, which we have already dealt with. The second is nearly unity when q is a considerable integer. The third has values as below—

$x =$	0	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{5}{6}$	1
$\sin \pi x/x =$	3·14	3·00	2·60	2·00	1·30	0·60	0·00

11. We see that x may be positive or negative: and neglecting the factor $2q/(2q+x)$, the maximum value of R_q will be at $x = 0$. It is the existence of this maximum which forms the basis of Professor Schuster's periodogram method, though there is this difference between his procedure and that here adopted: viz., he forms the integrals for complete periods of p, whereas we have formed them for complete periods of q. There will thus be a portion represented by x in one set of integrals which is not in the other, and we shall have a difference similar to that involved in neglecting the factor $2q/(2q+x)$. But this factor is very nearly unity, as already remarked, since $x < 1$ and q is a fairly large integer for any periodicity worth serious consideration. Hence the properties of the maximum will not be much affected.

12. Now it is clear that the maximum is a flat one. From $x = -\frac{1}{6}$ to $x = +\frac{1}{6}$, the value of $\sin \pi x/x$ only varies between 3 and 3·14, *i.e.* by about 5 per cent. in all: and accidental errors may swamp differences of this order. The proposition is only thus proved for a periodicity coinciding with a Fourier term: but though the relative *separation* of these terms is essential, the exact places where they occur are settled by the arbitrary choice of the whole number of observations. By adding or subtracting a few more observations we move the whole sequence bodily to right or left. Hence there can be no particular virtue at the place where a Fourier term occurs, and the proposition must be generally true.

13. We have above only tabulated values of x between 0 and 1; but it is easy to extend the series, and the following table

shows the approximate effect on neighbouring Fourier coefficients as p progresses from q to $q+1$. We shall take unity for the actual coefficient, and its effects on neighbouring terms are then as follows:—

TABLE I.

Fourier Terms.

Value of p	$q-3$	$q-2$	$q-1$	q	$q+1$	$q+2$	$q+3$ . . . .
$q+0$	·00	·00	·00	+1·00	·00	·00	·00 . . . .
$q+\frac{1}{6}$	+·05	+·07	+·14	+ ·96	−·19	−·09	−·06 . . . .
$q+\frac{1}{3}$	+·08	+·12	+·21	+ ·83	−·41	−·17	−·10 . . . .
$q+\frac{1}{2}$	+·09	+·13	+·21	+ ·64	−·64	−·21	−·13 . . . .

These values are approximate only, the factor $2q(2q+x)$ being neglected in order to make the table general. The method of formation, when once the coefficient under q is filled in, is simple. Suppose the values of q set off on a lever with fulcrum at the value of p: then any coefficient, regarded as a weight, would balance any other on the opposite side of the fulcrum, or be equivalent to any on the same side. When $p=q+\frac{1}{3}$, the fulcrum divides the interval q to $q+1$ into three parts. The coefficients on the positive side are as the reciprocals of 1, 4, 7, etc.; those on the negative side as the reciprocals of 2, 5, 8, etc.

14. If, then, there is an isolated periodicity, we can get the value of its period very precisely from the ratio of the coefficients where the change of sign occurs. But we see at once that the presence of another periodicity, even at a distance, may make trouble. Thus, suppose that there are two equal periodicities at $q+\frac{1}{2}$ and $q+3\frac{1}{2}$. The latter affects q and $q+1$ by + ·09 and + ·13 respectively, and thus converts the coefficients + ·64 and − ·64 into + ·73 and − ·51, altering the ratio of equality into one of 1·4. If the periodicity at $q+3\frac{1}{2}$ has the opposite phase, the alteration of period will be in the opposite direction.

15. If we know the approximate nature of the disturbing periodicities, we can make the requisite corrections before deducing the definitive value of any one of them: and when the periodicities are few and well defined, a process of approximation can clearly be arranged to overcome the difficulty indicated.

16. But "accidental" periodicities will also have their effects, and will disturb the actual period in an irregular manner. It may turn out that we have hitherto overestimated the precision with which a period can be fixed, as will be further illustrated when we come to an actual example.

17. Meanwhile we remark that by inverting the process above given, we can build up the particulars for any required periodicity which we may wish to examine, when we have the coefficients of the Fourier sequence before us. Let us take, for example, the coefficients for Sun-spots given in the table below in the neighbourhood of the 11-year period. They are as follows:—

$q=$	11	12	13	14	15	16	17	18
Period in years =	14·2	13·0	12·0	11·14	10·40	9·75	9·20	8·67
sines =	−5·5	+5·9	+14·7	−19·6	−13·2	+ 0·4	+6·5	+10·3
cosines =	+3·2	+4·8	+ 3·1	+15·8	− 4·8	+13·6	+3·7	+ 1·2

and let us suppose that we wish to find the coefficients for a periodicity $q = 13\frac{1}{2}$, midway between 12·00 years and 11·14 years: *i.e.* for 11·56 years. Beginning with the sines, the coefficient +14·7 under 13 gives a contribution $+14{\cdot}7 \times {\cdot}64$, as we see from Table I. above: the coefficient +5·9 under 12 gives $+5{\cdot}9 \times {\cdot}21$. In fact, we have simply to multiply these terms by the appropriate coefficients in the last line of Table I., getting

$$\left.\begin{array}{l} +14{\cdot}7 \times {\cdot}64 + \ 5{\cdot}9 \times {\cdot}21 - 5{\cdot}5 \times {\cdot}13 \\ +19{\cdot}6 \times {\cdot}64 + 13{\cdot}2 \times {\cdot}21 - 0{\cdot}4 \times {\cdot}13 \end{array}\right\} \text{Total, } +25{\cdot}3.$$

[The quickest way to multiply is to reserve the factor ·64 to the end, dividing successive terms by 1, 3, 5, 7, etc.]

The cosine coefficient is similarly found to be −7·5. Hence $S^2 + C^2 = 696{\cdot}4$, which is sensibly greater than the value 653·8 given by the coefficients for 11·14 years. We shall find other indications supporting a period of 11·5 years rather than that usually adopted (about 11·125 years). But we can see from the Fourier sequence why the latter has come to be adopted. If we look at the sines alone, the big terms −19·6 and +14·7 suggest a period nearly midway between them; but the cosines do not support this view. The change of sign in this series comes on the other side of $q = 14$. When different pieces of independent evidence thus conflict, we must resort to compromise, and the 11·125 year period is one possible compromise. But we really have a considerable choice, as may be seen not only from the figures given here, but from those tabulated by Professor Schuster. His periodogram near 11 years is

11·86	1951×10^3
11·75	2338
11·50	3700
11·25	4230
11·00	2724
10·75	742

And it would thus appear that the maximum lies somewhere between 11·25 and 11·50 years. But we need not at present examine the precise value further, until collateral evidence has been considered. (See § 35.)

(*c*) *The Fourier Sequence is convenient.*

18. The words "necessary" and "sufficient" are absolute and fundamental: it savours perhaps of anti-climax and of superfluity

to pass to relative words like "convenient" and "instructive." The words themselves may be ill chosen, but the ideas they stand for are very real, as may perhaps be gathered from the following brief sketch of an experience of periodogram work extending over several years.

19. Starting from the idea of examining as many periods as possible, without undue labour, the following process occurred to me. First break up the long series of observations into a series of short groups. Professor Schuster has drawn attention to the analogy between the length of the series of observations and the resolving power of a spectroscope. A long series means high resolving power, when a "bright line" would be well defined: a short series, low resolving power, when the line would be diffused. By breaking up the series into short lengths, we might, however, get a notion *whereabouts* the bright lines were to be suspected, and then, by recombination, get the necessary definition. In practice this might be attained as follows: take, say, a series of monthly rainfall observations and divide them into periods of thirteen months each. As a unit short group let us take five periods together: and let us determine S and C, the sine and cosine of the first harmonic, for each of these short groups. If there happens to be a periodicity of exactly thirteen months in the rainfall, then both S and C will tend to be constant from group to group. Accidental errors will interfere with the constancy, but the general mean values will persist in spite of the accidental irregularities. The formation of S and C for successive sets of five periods was called the *primary analysis*.

20. But there is probably no periodicity in *exactly* thirteen months, though there may be one in the neighbourhood. If so, then both S and C will show periodic changes. For instance, the well-known annual period is in the neighbourhood of thirteen months, though not very closely. In a single short group of five periods the maximum travels five months away, which is $5/12$ of a whole period, and is represented by $150°$. Thus S and C will change from group to group like $r \sin \theta$ and $r \cos \theta$, when

$$\theta = 150° \times n + \alpha,$$

where n has successive integral values. We can, therefore, examine *either* S *or* C, or both, for periodicity; with the advantage that we have replaced every 5×13 terms of the original series by a single term, which shortens the work considerably. This examination of S and C was called the "secondary analysis."

21. In this way much work was done: but in the secondary analysis it was not clear that anything had been gained over the graphical method originally suggested by Professor Schuster, which is simply to form θ from S and C by means of the relation

$$\tan \theta = S/C,$$

and then to plot the values of θ to see whether they indicate one or more straight lines. The multiplicity comes in through the

multiple values of θ, which differ by multiples of 360°. We must plot a sufficient number of these on the diagram to show lines of different slopes. Thus in the case of thirteen months already quoted there is pretty sure to be a line with such a slope that θ changes by 150° each time, indicating the annual term. But are there also other possible slopes? A reference to the example given below will perhaps make the point clearer (see § 31).

22. The next step is fairly obvious, though it was not taken for some time. If there are other possible slopes, they will appear more clearly if the big annual term is first cleared out of the way: and similarly if any other periodicity is established, it will clear the way for others to remove it, by some process or another, from the observations. When it was decided to proceed in this manner it was found that the removal of any periodicity for which there seemed to be reasonable evidence generally *did* lead to the revelation of another. The phenomenon was so general that attention was called to it at the International Congress of Mathematicians at Cambridge last year ("On Double Lines in Periodograms," *Proc. Fifth Internat. Cong. Math.*, vol. ii. p. 177). The explanation is very simple when we keep the properties of the Fourier sequence in mind, and will be referred to under the fourth head. For the present we are chiefly concerned with the accompanying difficulties that (*a*) whenever there are two or three periodicities thus closely associated, one seems to affect another so as to make identification of the elements of either very uncertain and almost arbitrary. This difficulty is specially prominent when we try to reconcile results obtained empirically near (say) $13\frac{1}{2}$ months from the primary analysis in 13 months, with those similarly obtained from a primary analysis in 14 months. (*b*) A second difficulty is that of determining when to stop, as the removal of any one term generally leaves something worthy of notice in the remainder.

23. Briefly, the purely arithmetical process at first adopted was very *long and tedious*: the graphical process suggested by Professor Schuster was shorter but more *indefinite*. The convenience claimed for the Fourier sequence is that it is short like the graphical process, and perfectly definite like the arithmetical. It combines in this way the advantages of both.

24. The computations may be thrown into a variety of forms. The straightforward method is to multiply each term by an appropriate sine or cosine. In the example of the Sun-spots there are 156 terms $= 4 \times 39$. Since the numerical value of a sine or cosine repeats four times in the whole circuit, there are 39 separate numerical values; or, if we start from zero, there will be 2 zeros, 2 unities, and 38 fractions. For the following work an old volume of Crelle's tables was partially sacrificed, the appropriate 38 tables being cut out and fastened on cards. The multiplications were then easily arranged by permutations of the cards. Some of the submultiples are of course easier than others. Thus for any submultiple $3n$, only thirteen cards were required. But these details need not be noted here.

25. The following are the results obtained from the analysis of the 156 yearly means for Sun-spots. It has been carried, for the sake of experiment, beyond the point at which a year is a suitable unit: the second half of the table is, indeed, open to this objection.

TABLE II.

Results of direct Fourier Analysis of 156 *Years of Wolf's Sun-spot Numbers.*

Divisor.	Period. y	S.	C.	R.	Divisor.	Period. y	S.	C.	R.
1	156·0	− 1·3	+ 2·1	2·5	28	5·58	− 2·9	− 3·6	4·6
2	78·0	+16·4	− 7·3	17·9	29	5·38	+ 3·9	+ 1·2	4·2
3	52·0	− 8·2	− 6·8	10·6	30	5·20	+ 0·1	− 1·1	1·1
4	39·0	+ 0·9	+ 4·1	4·2	31	5·03	− 2·8	+ 1·6	3·3
5	31·2	− 2·6	+ 1·6	3·1	32	4·88	− 3·5	− 1·4	3·8
6	26·0	+ 1·8	− 6·4	6·7	33	4·73	+ 0·2	+ 2·1	2·1
7	22·3	− 1·1	− 2·7	2·9	34	4·59	+ 1·5	+ 2·4	2·8
8	19·5	+ 3·6	+ 4·5	5·8	35	4·46	+ 2·1	+ 0·2	2·1
9	17·3	+ 1·3	+ 3·7	3·9	36	4·33	− 0·5	− 2·0	2·1
10	15·6	+ 1·8	− 5·9	6·2	37	4·22	− 1·0	− 1·3	1·7
11	14·2	− 5·5	+ 3·2	6·4	38	4·10	+ 2·5	+ 1·8	3·0
12	13·0	+ 5·9	+ 4·8	7·6	39	4·00	− 1·7	0·0	1·7
13	12·0	+14·7	+ 3·1	15·0	40	3·90	+ 0·8	+ 0·1	0·8
14	11·14	−19·6	+15·8	25·8	41	3·83	− 0·2	− 1·2	1·2
15	10·40	−13·2	− 4·8	14·0	42	3·72	+ 0·9	− 0·2	1·0
16	9·75	+ 0·4	+13·6	13·6	43	3·63	+ 1·7	+ 2·5	3·0
17	9·20	+ 6·5	+ 3·7	7·5	44	3·55	− 0·9	+ 1·3	1·6
18	8·67	+10·3	+ 1·2	10·4	45	3·47	− 0·3	+ 1·3	1·4
19	8·21	− 2·3	−10·3	10·6	46	3·39	+ 1·4	− 1·4	2·0
20	7·80	− 0·4	− 2·1	2·2	47	3·32	+ 0·5	− 0·2	0·6
21	7·43	− 2·6	+ 4·6	5·3	48	3·25	+ 0·3	− 1·2	1·3
22	7·09	− 5·3	+ 4·9	7·2	49	3·18	− 1·1	+ 0·1	1·1
23	6·78	− 1·9	+ 2·2	2·9	50	3·12	+ 1·6	− 0·7	1·8
24	6·50	− 0·1	+ 0·4	0·4	51	3·06	+ 0·7	+ 3·8	3·9
25	6·24	− 1·1	+ 0·2	1·1	52	3·00	0·0	+ 0·3	0·3
26	6·00	+ 3·3	+ 1·6	3·7	53	2·94	+ 0·5	− 0·7	0·9
27	5·78	− 4·4	+ 2·5	5·1	54	2·89	− 1·5	+ 0·4	1·6

26. One reason for continuing the table so far was to test the reality of the periodicity near 4·88 years, to which Professor Schuster has called attention. We see now by inspection what is the nature of the evidence. If there is but a single periodicity in this neighbourhood, the S and C series are apparently of the same sign, and we can add them together thus:—

Divisor	30	31	32	33	34
S + C	−1·0	−1·2	−4·9	+2·3	+3·9
Calc.	−0·7	−1·3	−5·0	+2·5	+1·0
Diff.	−0·3	+0·1	+0·1	−0·2	+2·9
S alone	+0·1	−2·8	−3·5	+0·2	+1·5
Calc.	−0·5	−0·9	−3·4	+1·7	+0·7
Diff.	+0·6	−1·9	−0·1	−1·5	+0·8
C alone	−1·1	+1·6	−1·4	+2·1	+2·4
Calc.	−0·2	−0·4	−1·6	+0·8	+0·3
Diff.	−0·9	+2·0	+0·2	+1·3	+2·1

The approximate period is inferred from the series for S + C to fall at $q = 32\frac{1}{3}$ or period 4·83 years, and referring to Table I. we see that the values at 32 are reduced in the ratio 0·83, so that the maximum values of S and C are

$$S = -5·0 \qquad C = -1·9 \qquad R = (S^2 + C^2)^{\frac{1}{2}} = 5·6.$$

This seems to give a satisfactory determination of the elements, the only doubt being whether the coefficient is large enough to be real.

27. There is a larger one between 28 and 29. Here again S and C are sympathetic in sign, and we have

Divisor	26	27	28	29	30	31
S + C	+4·9	−1·9	−6·5	+5·1	−1·0	−1·2

Here we get no help from surrounding terms, which are all inconsistent except 27. The period is nearly midway between 28 and 29, though nearer 28 as the figures stand. If, however, we first correct for the 4·83 year period, the above numbers become

+5·2 −1·6 −6·1 +5·6 −0·3 +0·1,

which brings it nearer midway, say 5·70 years.

28. The further discussion of these numbers will be taken later. We are at present dealing with the *convenience* of the Fourier sequence; and the convenient way in which it lends itself to identification of the true periods has been sufficiently illustrated. We now come to a different point, the adaptation of methods other than the straightforward process above used, which becomes tedious and unsuitable when the number of terms is large. Thus for the investigation of terms of short period we must take a smaller unit than one year—let us choose 3 months. Then we shall now have 4 × 156 terms, and should require 156 separate cards. Of course we could divide our material into 4 sets of 156 terms each, and then proceed as before: but this is to lose "resolving power." The submultiples ought really to be 4 times as close as those to which we should be led by a set of 156 terms, and even then we should have to do the work 4 times. It is better to use an indirect process.

29. An outline of the process proposed has already been given,

and we will now give precision to it by an example. Let us take that already considered, Professor Schuster's period near 4·8 years. Twenty units of 3 months each make 5·0 years, 19 units make 4·75. We might take either, but we will take the former. If we take 5 periods together = 25 years, we do not eliminate the effects of the 11-year period very well; and if we take 4 periods together = 20 years, we are not much better off: but the latter will give us more separate groups, so we will choose that.

30. Forming S and C for successive groups we get—

TABLE III.

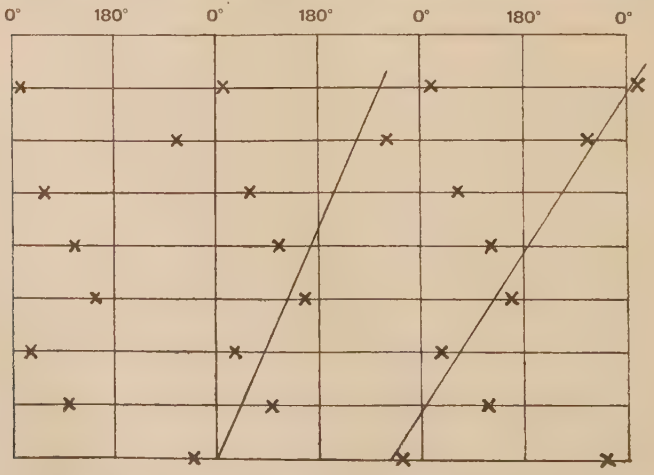
Analysis for Twenty Terms of Three Months each.

S.	C.	R.	θ.	(1).	(2).	(3).	θ_1.	θ_2.	θ_3.
+ 0·5	+ 3·8	3·8	8°	− 5°	+ 18°	+ 41°	3	26	49
−16·8	+ 4·3	17·4	284	−14	+ 55	+124	270	339	48
+ 9·5	+ 3·9	10·3	68	−24	+ 92	+208	44	160	276
+ 6·0	− 3·2	6·9	118	−33	+129	+291	85	247	49
+ 7·9	−10·1	12·8	142	−43	+166	+ 15	99	308	157
+ 8·8	+12·0	14·9	36	−52	+203	+ 98	344	239	134
+ 8·7	− 6·4	10·8	127	−62	+240	+182	65	7	309
− 4·2	+ 2·3	5·0	299	−71	+277	+265	228	216	204

31. The graphical process of Professor Schuster would work with the phases θ given in the fourth column, deduced from the equation

$$\tan\theta = S/C.$$

If these are plotted on a diagram as below, it is easily seen that a straight line could be drawn through them somewhat as shown.



32. But there is considerable latitude available in the drawing, as indicated by two alternative lines. The process now advocated

removes this uncertainty by providing us with definite slopes to be examined, namely, those corresponding to the definite periods of Table II. We confine our attention chiefly to periods near 4·8 years and therefore *less* than five years (*i.e.* lines sloping downwards to the left). And Table II. tells us that we are to consider the periods 5·033, 4·875, 4·726 years, which will suffice for illustration, though others could easily be added. Let the phase diminish by x degrees in four periods. Then,

$$5\Big/\Big(1+\frac{x}{4\times 360^\circ}\Big)=5\cdot033,\quad \text{or}\quad 4\cdot875,\quad \text{or}\quad 4\cdot726,$$

$$\therefore\ x=-9^\circ\cdot4,\quad \text{or}\quad +37^\circ\cdot0,\quad \text{or}\quad +83^\circ\cdot4.$$

The difference between each pair is 46°·4, and in eight groups this accumulates to 360° + 11°·2—rather over a complete cycle. In fact we have used a few more observations than those of the selected 156 years to complete the last group. But this ought not to make any serious difference.

33. In Table III. the successive additions to θ are shown in the columns headed (1), (2), (3), and the results of adding them in columns θ_1, θ_2, θ_3. It will be noticed that the first term added is half the common difference. This is to reduce to the *beginning* of the first group, as a common epoch. We have now to multiply the column R by the sines and cosines of θ_1, θ_2, θ_3, as in Table IV.

TABLE IV.

R sin θ_1.	R cos θ_1.	R sin θ_2.	R cos θ_2.	R sin θ_3.	R cos θ_3.
+ 0·2	+ 3·8	+ 1·7	+ 3·4	+ 2·8	+ 2·5
−17·4	0	− 6·3	+16·2	+12·9	+11·7
+ 7·1	+ 7·4	+ 3·5	− 9·7	−10·2	+ 1·0
+ 6·9	+ 0·6	− 6·3	− 2·7	+ 5·2	+ 4·6
+12·7	− 2·0	−10·1	+ 7·9	+ 5·0	−11·8
− 4·2	+14·3	−12·8	− 7·7	+10·7	−10·3
+ 9·8	+ 4·5	+ 1·3	+10·7	− 8·4	+ 6·8
− 3·7	− 3·3	− 2·9	− 4·1	− 2·0	− 4·7
+ 1·5	+ 3·2	− 4·0	+ 1·8	+ 2·1	0·0

34. The means of the columns are given at the foot of each, and if we now arrange them as before we have:—

Period	5·033	4·875	4·726
S	+1·5	−4·0	+2·1
C	+3·2	+1·8	0·0
S − C	−1·7	−5·8	+2·1
Calc.	−1·2	−6·0	+2·0

It would seem that S and C are now of opposite signs. [This is not inconsistent with § 26, for the epoch of reference there is the middle of the first *year*, while we now take the middle of the first

three months.] Forming S – C the true period assigned by the figures is approximately one-quarter of the way from 4·875 to 4·726 years, *i.e.* $4{\cdot}875 - 0{\cdot}149/4 = 4{\cdot}838$ years, agreeing sensibly with the result of § 26. The sines are responsible for about $\frac{2}{3}$ and the cosines for $\frac{1}{3}$ (numerically) of S – C. Thus the coefficient of the term is

$$(\tfrac{4}{9} + \tfrac{1}{9})^{\frac{1}{2}} \times 6{\cdot}0/0{\cdot}90 = 5{\cdot}0,$$

which is not far from 5·6 of § 26.

35. But, having thus illustrated the manner in which the Fourier sequence may be obtained in an indirect manner, even when the number of terms is large, we shall not pursue the analysis further at present. The complete examination of terms of short period must be reserved for another paper. It has already been commenced, and some interesting results obtained. There is, for example, a marked periodicity near 2·88 years which may be the quarter harmonic of the 11-year period, thus supporting the value $4 \times 2{\cdot}88 = 11{\cdot}52$ years suggested in § 17. But the point is so important that it must be fully examined.

(*d*) *The Fourier Sequence is Instructive.*

36. In gravitational theory we are familiar with the notion of a number of perfectly crisp periodicities, each with an accurately defined period, and a coefficient which is often related to other coefficients in a precise manner. If all the phenomena of Nature were of the same type as gravitation, the credentials of a method such as that now under consideration would begin and end with its being "necessary and sufficient" as above. It may be that scientific public opinion will refuse to recognise any other type of periodicity: but the study of the periodogram and similar methods has led me to believe that the existence of other types is at any rate a possibility, and that the evidence for the possibility is instructively displayed by the Fourier sequence. The evidence is as yet only incipient, and closer scrutiny may destroy its validity. But there are at any rate some facts which are worthy of more attention than they have so far received.

37. The main feature to which I wish to direct attention has been already mentioned in § 22, and was in another form brought before the International Congress of Mathematicians at Cambridge in 1912. I read a note on "Double Lines in Periodograms," the point of which was that when a valid periodicity is detected and eliminated from the observations, we often become aware of another in the immediate neighbourhood. One or two examples were given, but many more might have been cited: and we need go no further than the Sun-spot results given above to find good instances. Suppose we detect and remove the fourteenth harmonic of Table II., period 11·14 years: then we shall become aware of either the thirteenth or the fifteenth or both. It may be said that this is because we have not rightly identified the period of 11·14 years. But it has already been shown in § 17 that it is

difficult to identify any completely satisfactory period: whatever we adopt, we leave much unexplained: *i.e.* whatever single periodicity we remove, we uncover one or more in the neighbourhood.

38. When the above-mentioned paper was read, Sir Joseph Larmor asked under what conditions these separate periodicities could be distinguished: and I replied (to his satisfaction): When one runs through at least one period relatively to the other during the available observations. I might have added that cases where the difference was one period within close limits were not uncommon, and had already arrested my attention: but at the time no explanation of the phenomenon had occurred to me. The method of the Fourier sequence is suggestive in this connection. If two or three successive Fourier coefficients are all large, the suppression of one will direct attention to the next on one side or the other.

39. The main question may thus be put into this form: *Why should two or three successive Fourier coefficients all be large?* I venture to suggest that hitherto this question has not been faced. When the observations have failed to reveal a single definite periodicity, there has been a vague feeling that the series was not long enough to separate the different periodicities, and that we must await more observations. But this view is really untenable: from the very nature of the Fourier sequence each coefficient is independently determined, and the series, whatever its length, is always long enough to separate its own harmonics. If we have merely failed to identify the correct period, the simple relationships between the successive coefficients shown in §§ 8-14 will enable us to set this right: but we may find, and we do often find, that even then the neighbouring periodicities are still noteworthy. Why?

40. One line of explanation is suggested by the consideration of the annual period in meteorological phenomena: say the Greenwich rainfall. There is a large coefficient for the period 12 months, but there are also respectable coefficients for the Fourier terms on opposite sides of 12 months. Here we have no doubt about the exact period of the main periodicity, and we can remove it without any margin of doubt. What is the significance of those that are left? Are there other identifiable periodicities near one year—possibly the sidereal year, say, instead of the tropical; or the anomalistic year?

41. One possible reply at any rate is in the negative. It may be urged that though the tropical year is perfectly definite, its manifestations in the Greenwich rainfall are only rough, and that this is quite sufficient to provide the adjacent lines, which are entirely spurious. That the annual effect on the rainfall is only roughly annual is obvious to the most casual observation: the maxima vary largely not only in amount but in time of occurrence. Let us take the simplest possible case: in three successive years let the maxima fall in September, August, and September

respectively. The first pair will tend to return the period as 11 months, the second pair as 13 months: so that we shall get indications in the periodogram on opposite sides of the true period, due to the single displacement of one maximum. If the middle maximum fell in October instead of August we should also get this pair of contributions. In fact, any displacement of a maximum gives us a pair of contributions on opposite sides of 12 months, the separation from 12 months depending on the amount of displacement. Large displacements will be unusual, small ones more numerous; and hence the pairs of contributions will cluster round 12 months somewhat in the manner of a probability curve. Now, the Fourier sequence gathers these up into a series of definite steps: each Fourier term may be regarded as the integration of the terms in its immediate neighbourhood, but its range is limited. The annual term can only gather up the periods close to it, leaving others to the Fourier terms on opposite sides of it. From this point of view these terms are therefore not new or different, but only manifestations of the annual term in another form. There is only one real periodicity, the tropical year, which is, however, disturbed by "accidental" causes, and therefore appears as a series of terms. Similarly, there may be only one real Sun-spot period (though we are not yet able to identify it independently as the tropical year can be identified), which is disturbed by accidental causes and therefore appears as the extensive series of terms indicated in Table II. from (let us say) divisor 10 to divisor 19. This is the view (if I understand him rightly) taken by Mr. Maunder, who stoutly denies the existence of any other periodicities but one of about 11 years.

42. I have sketched this view as sympathetically as I can, because I think it must be very carefully considered. But it is not my own view, and it has at least the disadvantage of tending to stifle investigation. It may be that the periodicities which affect a number of consecutive Fourier terms are merely single and simple periodicities disturbed by "accidental" causes, and that the study of these "accidents" by any form of the periodogram method is unprofitable, if not even based on fallacy. But it must be remembered that the disturbances are not necessarily accidental: their effect would be of the same general character if they were strictly periodic. If the rainfall maximum fell in August every 11 years, the reasoning of § 41 would still apply, and we should still get a pair of terms on opposite sides of the annual. It is true that we should also get a periodicity of 11 years, which would materially help our interpretation: but the case might not be so simple as this, and the interpretation might only come from prolonged study of all the information. If this is so, then anything which puts the information into such a shape as to facilitate that study is of value. I claim for the Fourier sequence that it is "instructive," for the reason that it seems to me to put the available information into the handiest shape for discussion which has as yet come within my experience.

43. Two concrete instances of "instructiveness" may be given in illustration. The first must be partly surmise, as has already been indicated in § 35. It seems possible that the precise value of the 11-year Sun-spot period (assuming it to have a precise value such as the tropical year) may be indicated by one of its sub-multiples. In many respects the Fourier sequence offers no advantages over the Schuster periodogram for this purpose: the one point of difference is that the Fourier sequence points directly to the analysis of the whole series of observations, while Professor Schuster has allowed an economy of labour by using only a portion of the series for terms of short period. This economy is in no way essential to his method, and can be dispensed with, especially when an important term has been localised approximately. But it seems worthy of remark that if we analyse the whole series for all terms, however short, the labour may be repaid in increased accuracy of the long periods through their harmonics. If L be the length of the series, the period of the nth harmonic is L/n, and of the $(n+1)$th is $L/(n+1)$. The difference between these two periods is $L/n(n+1)$. Now, the accuracy with which we can identify a period has in all probability some relation to this interval: let us assume that it can be measured by the xth part of this interval and put $L/x = M$, so that $M/n(n+1)$ is our measure of accuracy. The accuracy of the rth harmonic will be $M/n(n+1)r^2$, which gives the period of the fundamental tone with an accuracy

$$rM/n(n+1)r^2 = M/n(n+1)r.$$

Thus as r increases we get a more and more precise determination of the fundamental tone. But we sacrifice this advantage if we save labour by using only $1/r$ of the whole series L.

44. The second example is from the theory of the movement of the Earth's Pole. It is known that there is a 14-monthly periodicity and also a 12-monthly. In the course of further analysis of the observations, Chandler found (*A.J.*, No. 522) a period of 13 months; and finally (*A.J.*, No. 523) he was almost reluctantly driven to announce another of 15 months. Now, this sequence of 12, 13, 14, 15 months seems to demand explanation, and one simple explanation can be given in terms of the Fourier sequence. If these are approximately consecutive terms of a Fourier sequence, the whole length L of the period of observations must be about

$$15 \times 12 \text{ months} = 13 \times 14 \text{ months very nearly,}$$

i.e. about 15 years. This is *of the same order* as the length of time for which the movements of the Pole have been under accurate observation; and though it may not be the actual length of time used by Chandler, it may in some way (owing to methods of weighting, etc.) be the equivalent of his length of time. Successive Fourier periods would then be 12·0, 12·9, 13·8, 15·0 months (using divisors 15, 14, 13, 12). Now, we have seen that by eliminating divisors 13 and 15 we are apt to uncover divisor 14:

i.e. when the best possible allowance is made for the annual and 14-monthly terms we should naturally reveal a 13-monthly term, if it had any sensible value. And supposing all these three terms to be removed, then the next one of 15 months (divisor 12) would appear. [The *next* will show up in preference the *next but one* owing to its proximity: as we depart further and further from any periodicity under discussion, there is a rapid reduction of amplitude.] It seems possible that Chandler has in some way hit upon successive Fourier terms, or their equivalent. This would not destroy the validity of any of his terms: but it would subject them to the same scrutiny that must be given to all such associations of several Fourier terms. Some such associations or groups will doubtless turn out to be accidental in character, but we may fairly expect that others will give us clues to new knowledge. I do not (at present at any rate) see how the periodogram or the Fourier sequence can, of themselves, distinguish between the two classes of groups, or tell us whether there are two classes. This must come from independent information which our clues lead us to consult.

45. One final remark may be adventured on the relation of the Fourier sequence to prediction. Let us represent our observations, expressed in Fourier sequence, as

$$\text{Experience} = \Sigma A_n \cos n(\theta - B_n)$$

where n is the divisor of the whole length of time. Each term represents an event which has happened successively n times within our past experience, and for all we know may have been happening in the same regular way during the time when we were not watching. The probability that it will go on recurring in the same way indefinitely may be denoted by $\phi(n)$. To assume that it is a simple function of n is of course a large assumption, only made here for the purposes of illustration. In that case the best prediction we can make for the future would be

$$\text{Prediction} = \Sigma \phi(n) A_n \cos n(\theta - B_n).$$

46. One direction in which the above assumption is probably incorrect is that $\phi(n)$ is also a function of A_n. Thus, when A_n is small, the periodicity may be entirely due to errors of observation: and in other cases these are included to a partial extent. We had therefore better write

$$\text{Prediction} = \Sigma \psi(n, A_n) A_n \cos n(\theta - B_n).$$

But if this is approximately the best we can do, there is one important corollary for the groups of Fourier terms. Suppose for simplicity that two consecutive Fourier terms are of equal amplitude. Then, since $\psi(n, A_n)$ cannot vary rapidly with n, and A_n is the same, we have

$$\psi(n, A_n) A_n = \psi(n+1, A_{n+1}) A_{n+1}$$

approximately: so that if two equal terms occur in experience, we shall have two nearly equal terms in prediction. The extension of

the principle to groups is obvious. Applying the principle to an example, let us take the Sun-spot results of Table II. To make a prediction for the future we should use *not* a single periodicity near $n = 14$, *but a group* of terms, roughly similar to the group $n = 13, 14, 15, 16$. Since the coefficient R for $n = 14$ is distinctly larger than the other three, we should probably give it more weight, and we may even devise a formula for the weight. Further, since $\phi(n)$ or $\psi(n)$ certainly increases with n, we should give increasing weight to successive terms. But the four terms as they stand will be a rough approximation, and will furnish a much better prediction than any *single* term.

47. There is, however, another direction in which our first assumption may be incorrect which is much more difficult to deal with. We have tacitly assumed the separate members of the Fourier sequence to be independent events, and this assumption may not be true. In that case our function ϕ or ψ will not be a simple function of the characteristics of one member alone, but will be a multiple function of those of several. At the present moment I do not see how better to elucidate this possibility than by accumulating material from actual observations and studying it. There *may* be a purely theoretical method of investigation: but even then the accumulated material will be valuable as a test. In the case of the ordinary theory of errors such tests are welcome: and in this case of periodicities, which has some similar characteristics but is certainly more complex, comparison with the results of observation should be at least as welcome. Hence it seems desirable to submit to the analysis of the strict Fourier sequence as many representative series of observations as possible.

48. The theorems of § 8 take the following form when the number of observations is finite. We assume a single periodic term

$$\sin(r\theta + \alpha)$$

where r is the number of the observation in the whole series of n observations, and θ is the angle by which the phase of the true period increases between one observation and the next. Thus r, which increases regularly with the time, corresponds to the t of § 8, and θ to p. To form the Fourier coefficients S and C, we find

$$\tfrac{1}{2}n\mathrm{S} = \sum_{r=0}^{r=n-1} \sin(r\theta + \alpha)\sin r\phi, \qquad \tfrac{1}{2}n\mathrm{C} = \sum_{r=0}^{r=n-1} \sin(r\theta + \alpha)\cos r\phi$$

where $n\phi = 2\pi q$, and q is an integer as in § 8.

By the ordinary processes of summation, we find

$$\mathrm{S} = \cos(\tfrac{1}{2}n\theta + \alpha) \,.\, \sin\phi \,.\, \sin\tfrac{1}{2}n\theta/n \,.\, \sin\tfrac{1}{2}(\theta + \phi)\ \sin\tfrac{1}{2}(\theta - \phi),$$
$$\mathrm{C} = \sin(\tfrac{1}{2}n\theta + \alpha) \,.\, \sin\theta \,.\, \sin\tfrac{1}{2}n\theta/n \,.\, \sin\tfrac{1}{2}(\theta + \phi)\ \sin\tfrac{1}{2}(\theta - \phi).$$

When $\theta = \phi$, these become $\mathrm{S} = \cos\alpha$, $\mathrm{C} = \sin\alpha$, as they should.

When n is very large, we have

$$\theta/p = \phi/q = 2\pi/n,$$

and on substituting these values and putting n infinite we get the formulæ of § 8.

Summary.

Professor Schuster has urged that for regularly recorded elements such as magnetic variation or rainfall a "periodogram" should be formed by calculating S and C, the coefficients of all possible periodic terms, and displaying in a diagram the values of $S^2 + C^2$.

The suggestion of the present paper is that the calculation of S and C be restricted to periods which are the exact submultiples of the whole length of time during which available observations have been made, choosing the exact value of this whole length with regard to convenience, within certain limits.

From this "Fourier sequence" the values of S and C for any other intermediate periods can be quickly and simply inferred (§§ 8 to 17). Hence this sequence is necessary and sufficient.

That it is also convenient in practice is shown by an example from the Sun-spot numbers. The early terms of the sequence were calculated by the direct method (§§ 25 to 27). But submultiples of high orders are best calculated by an indirect method, §§ 28 to 35.

In §§ 36 to 47 the significance of the sequence is discussed.

Further Note on the Possibility of Refraction by the Solar Atmosphere. (*Papers of the I.U.S.R. Computing Bureau, No. XI.*) By R. S. Capon.

(*Communicated by Prof. H. H. Turner.*)

Shortly after the publication of a "Note on the Possibility of Refraction by the Solar Atmosphere" (*M.N.*, March 1913), Professor Kempf drew my attention to a discussion on the subject of solar refraction which took place in 1865 and the two or three subsequent years. The chief participants were Spoerer, Secchi, and Faye. That a correction is to be applied to the observed position of a spot to find its actual position, appears never to have been in question. The argument centred on the cause to which the displacement was to be assigned.

The variation of the apparent angular velocity of the same spot at different places on the sun's disc was early noticed, and a possible dual explanation (the radius and refraction effects of the above note) was recognised. Faye introduced the term "depth-parallax" ("parallaxe de profondeur," *Comptes Rendus*, vol. 62, p. 364) to designate the former. Peters showed (*Ast. Nach.*, No. 1696, cols. 241–252) that, if the Cassini refraction formula were supposed to hold on the sun, the coefficients due to the two explanations would enter as a sum, and therefore could not be separated by direct methods. His formula is

$$d\rho' = (\beta + p/\mathrm{R}) \tan (\rho + \rho')$$

where ρ' is the heliocentric angular distance of the spot from the centre of the sun's disc, ρ is the corresponding geocentric distance, p the depth of the spot below the photosphere, and R the radius of the sun. Faye (*Ast. Nach.*, No. 1717, cols. 193–200) added a term $d\mathrm{R}''/\mathrm{R}''$ to the coefficient of tan $(\rho+\rho')$, where R″ is the apparent radius of the sun, and $d\mathrm{R}''$ the difference between the apparent and actual radii. He considers that the actual diameter of the sun can be discovered from transits of Mercury, and by other methods, so that $d\mathrm{R}''/\mathrm{R}''$ may be obtained by actual observation. He also suggests a means of separating the other terms (*Comptes Rendus*, vol. 61, p. 865) by measuring the distances between the margins of the nucleus and penumbra for different positions of spots on the disc. This eliminates β. Modifications of this method are used by Secchi, Peters, and Lohse. In Table A the values of k, the coefficient of tan $(\rho+\rho')$, and its components, are given in radians.

TABLE A.

	β	$d\mathrm{R}''/\mathrm{R}''$.	p/R.	k.	Reference.
Peters		0·0080	0·0007	0·0087	*Ast. Nach.*, No. 1696, cols. 241–252.
Faye	zero	0·0019	0·0053	0·0072	*Comptes Rendus*, vol. 64, p. 209.
Spoerer		...	...	0·0022	*Pub. der. Ast. Gesell.*, xiii. part ii.
Lohse		...	0·0027	...	*Ast. Nach.*, No. 1976, cols. 113–118.
Capon		...	...	0·0058	*M.N.*, March 1913.

In addition to the values tabulated above, Faye (*Comptes Rendus*, vol. 61, p. 1082) found values between 0·005 and 0·009 for k from three spots; and Secchi (*Comptes Rendus*, vol. 63, part ii. p. 163), from observations on a few spots, states that β is negligible.

The values of k, Spoerer's excepted, are fairly concordant. Taking 0·0060 as the mean of the four values expressed as above in radians, the equivalent in heliocentric degrees is 0°·344. This mean happens to be the value adopted for k in my former note. Table VII. of the note therefore gives the ratio of the observed to the actual angular velocities at the equator, and the divergence is seen to be by no means negligible. There will be many unexplained anomalies in the movements of spots (for example, that cited in the note on page 367) so long as this correction is not applied. I am informed by Professor Kempf that at Potsdam the correction has been made since 1880, but it has never been adopted, apparently, at Greenwich.

Let us now turn to the effect on the *latitude* of a spot. Adopting the mean value 0·0060 for k, the theoretical values of $\frac{1}{2}(\Delta_{60}+\Delta_{-60})-\Delta_0$ (where Δ_l is the N.P.D. when the spot is at longitude l measured from the central meridian) have been calculated, and are exhibited in Table B, together with the observed values of this quantity derived from Table II. of the note. (Note that, as in Table II., the south latitude numbers have been interchanged with the corresponding numbers for the north latitudes in

the case of the September months, so that the latitudes as they stand may have the same relation to the latitude of the earth on the sun for March and September.)

TABLE B.

Hel. Lat.	+25	+20	+15	+10	E	0	−10	−15	−20	−25
Feb. Mar. Apr.	$+0°27_{3}$	$+0°19_{22}$	$-0°05_{35}$	$-0°02_{52}$	$-0°07_{49}$	$0°00_{65}$	$-0°05_{57}$	$+0°01_{52}$	$-0°13_{13}$	$-0°16_{10}$
Aug. Sept. Oct.	$+0°28_{4}$	$-0°03_{26}$	$-0°01_{51}$	$-0°09_{22}$	$-0°03_{62}$	$+0°03_{56}$	$-0°14_{73}$	$-0°17_{29}$	$-0°22_{21}$	$-0°19_{8}$
Weighted Means	$+0°28_{7}$	$+0°07_{48}$	$-0°03_{86}$	$+0°01_{74}$	$-0°05_{111}$	$+0°01_{121}$	$-0°10_{130}$	$-0°05_{81}$	$-0°14_{34}$	$-0°17_{18}$
Calculated Values	$-0°04$	$-0°04$	$-0°04$	$-0°04$	$-0°04$	$-0°04$	$-0°04$	$-0°04$	$-0°05$	$-0°06$

It is clear that here the parallax-refraction effect, if it exists, is masked by variations due to other causes. There are three causes which can give rise to an apparent motion in latitude :—

(1) Parallax-refraction.
(2) Error in the adopted position of the sun's axis.
(3) Proper motions of the spots.

Of these (3) will probably give rise to effects symmetrical with respect to the solar equator, while (1) and (2) will exhibit asymmetry. The effect of (2) can easily be shown to be $-\frac{1}{2}Y$ for the March months, $+\frac{1}{2}Y$ for the September months, where Y is the angular displacement of the true axis from the adopted axis towards heliocentric longitude 90°. (See Carrington, "Observations of Solar Spots," p. 232.) Since the parallax-refraction coefficient, as exhibited in the last row of Table B, is almost constant, and also changes sign for the September months, the two effects cannot be separated by this means. On the assumption of symmetrical proper motions, (3) may be eliminated by adding the weighted means of corresponding north and south latitudes. The results for the weighted means of Table B are contained in Table C.

TABLE C.

+25	+20	+15	+10	0
−0°·04	−0°·02	−0°·04	−0°·06	−0°·02

These numbers represent the values of $-\frac{1}{2}Y$ + refraction-parallax coefficient. Their mean is −0°·04, which is exactly the mean of the theoretical values of the refraction parallax coefficient alone. If therefore we are content to adopt the position of the axis defined by Carrington, Y is zero and the numbers lend additional support to our hypothesis. Otherwise, we must await a redetermination of the position of the axis before these additional data can be applied.

It should be noted, however, that a determination of the elements of the axis from the motions of spots which does not allow of the elimination of the displacements due to parallax-refraction—for the evidence of Table A seems to leave little reason to doubt the real existence of these displacements—will be valueless for this purpose, for the value of Y determined will include the coefficient.

On the Reduction of Polar Trail Plates.

By Henry Norris Russell, Ph.D.

§ 1. *Introduction.*—The importance of photographs of the region immediately surrounding the pole, with multiple exposures taken at intervals during the night with a fixed instrument, was first pointed out by Pickering. If the telescope and its accessories remain really fixed, such plates should give a very accurate determination of the position of the pole, relative to the neighbouring stars. Jacoby * has given a detailed discussion of the theory of such plates, with numerical examples, using polar co-ordinates throughout. The recent proposals of Turner † for the determination of fundamental star places by photography add interest to this problem, and observations with a telescope of 10 inches aperture and 100 inches focal length, mounted firmly upon a pier in a basement room of nearly constant temperature, with access to the polar sky through a window of plane parallel glass, are now being regularly made at the Allegheny Observatory.‡ It may therefore be appropriate to present a method for the treatment of such plates, using rectangular co-ordinates of the stars, which it is hoped may have some advantages as regards simplicity and rapidity of computation.

§ 2. *Statement of the Problem.*—The observational data presented by such plates are the rectangular co-ordinates x, y of the star images, referred to some arbitrary system of axes, fixed with respect to the plate. We will assume that these are free—or have been freed—from errors of the measuring instrument, and also that the plane of the plates is perpendicular to the optical axis of the telescope and that the latter is sensibly parallel to the Earth's axis. We have to find, as objects of our study, the co-ordinates X_0, Y_0 of the stars, referred to some system of rectangular axes fixed (so far as is possible), with reference to the stars themselves, and the co-ordinates P, Q of the celestial pole, with reference to this same system: and must also determine, as by-products, the co-ordinates p, q of the pole on the plate, with respect to the system x, y, and certain plate-constants, which by proper adjustments may be made very small. These quantities will vary at random (within certain limits) from plate to plate, while P, Q change but slowly, and in accordance with known laws, and X_0, Y_0 are practically invariable.

Changes in X_0, Y_0 may arise from annual parallax (which is almost sure to be insensible) or proper motion (which may be determined by comparison with earlier photographs of the region,

* "Photographic Researches near the Pole of the Heavens," *Bulletin of the Imperial Academy of Sciences of St. Petersburg*, series 5, vol. ix., No. 1, pp. 41–82, 1898.

† *Monthly Notices*, vol. lxxi. pp. 416–438, 1911.

‡ *Publications of the Astronomical and Astrophysical Society of America*, vol. ii. p. 106, 1911.

and corrected for, if necessary). Changes in P, Q will arise from precession and nutation, which change the position of the pole; from the mean proper motion of the group of reference stars employed, which alters the standard of reference, but is likely to be very small; and from aberration. This changes the apparent positions of the stars by different amounts, but, for a small field, is equivalent to a change of centering and an alteration in scale-value (which may be shown to be proportional to the component of the Earth's velocity in the line of sight). Since our system of reference is to be fixed with respect to the stars, we must apply to P, Q the reversed aberration for a star at the pole.

§ 3. *Motion of the Pole.*—The effects of precession, nutation, and aberration upon the co-ordinates of the pole may be found at once from the Besselian star-numbers given in the nautical almanacs. With the notation of the American Ephemeris, we have *

$$P - P_0 = 20''{\cdot}046A + D\,;\ Q - Q_0 = -B - C \quad . \quad (1)$$

where P_0, Q_0 are the co-ordinates of the mean pole for the beginning of the year, and, for the moment, P is supposed to be measured toward 0^h and Q toward 6^h. The daily motion of the pole, when the aberration is included, varies from about $0''{\cdot}24$ to $0''{\cdot}42$, so that its changes during the night cannot be neglected in refined work. We will therefore set

$$P = P' + \delta P\,;\ Q = Q' + \delta Q \quad . \quad . \quad . \quad (2)$$

where P', Q' are the values for some definite epoch during the observations, and the small changes δP, δQ can be calculated by (1).

To allow for the change of scale-value due to aberration, we may set

$$X = X_0(1 - \epsilon)\,;\ Y = Y_0(1 - \epsilon) \quad . \quad . \quad (3)$$

where ϵ may be found by dividing the quantity i, given among the "independent star-numbers" by 206265, and never exceeds $\pm 4 \times 10^{-5}$.

We will now suppose that the origin of X_0, Y_0 has been chosen very near the pole, so that P, Q are small. Unless the observations extend over several years, they may always be made less than $40''$. We will also suppose that the axis of X_0 is directed towards the right ascension α_0, and that of Y_0 towards $\alpha_0 + 6^h$, leaving α_0 arbitrary for the present.

§ 4. *Refraction.*—Passing now to the actual plate, we may define an ideal system of co-ordinates x', y' upon it, such that the origin is at the projection of the pole, x' is measured downward, parallel to the projection of the meridian, and y' horizontally towards the west. If it were not for refraction, we would have, at the sidereal time θ,

$$\left.\begin{aligned} x' &= (1-\epsilon)\{(X_0 - P)\cos(\theta - \alpha_0) + (Y_0 - Q)\sin(\theta - \alpha_0)\} \\ y' &= (1-\epsilon)\{(Y_0 - Q)\cos(\theta - \alpha_0) - (X_0 - P)\sin(\theta - \alpha_0)\} \end{aligned}\right\} \quad (4)$$

* This can be proved at once by considering the changes in the apparent declinations of stars near the pole.

If x'', y'' are the co-ordinates, referred to the same axes, but influenced by refraction, we will have,*

$$\left.\begin{aligned} x'' &= (1-\beta)x' + \beta\cot\phi - \beta x' \cot^2\phi + \beta\cot\phi(y'^2 + x'^2\operatorname{cosec}^2\phi) \\ y'' &= (1-\beta)y' \end{aligned}\right\} \quad (5)$$

where β is the constant of refraction and ϕ the latitude.

The first terms represent a change in scale-value, which is the only effect in y''. The second term in x'' corresponds to a bodily displacement of the field, which for latitude 40° is about 70″. The last two terms in x'' represent a distortion of the field. For a star 1° from the pole (if $\phi = 40°$) they can amount at most to 1″·44 and 0″·05 respectively. The term involving y'^2 never exceeds 0″·02. If this is neglected, the others may be put in a single-entry table with argument x'.

The variations of β with the meteorological conditions are of great importance in the term $\beta\cot\phi$, and may sensibly affect the distortion terms. If the stars are of different spectral types, it is probable that the effective value of β will vary from star to star. We may therefore set

$$\beta\cot\phi = r_0 + \delta r + s$$

where r_0 is the mean value for the night, δr the difference between the value computed from the conditions at the time of exposure and this mean, and s is the unknown correction for colour, constant for each star.

If Δ represents the correction for refractional distortion, we may now write

$$\left.\begin{aligned} x'' &= x'(1-\beta) + r_0 + \Delta + \delta r + s \\ y &= y'(1-\beta) \end{aligned}\right\} \quad . \quad . \quad (6)$$

§ 5. *Motion of the Telescope.* — The co-ordinates actually measured on the plate may differ from the ideal system in centering, orientation, and scale-value; whence

$$\left.\begin{aligned} x &= (1+a)x'' + by'' + p \\ y &= (1+a)y'' - bx'' + q \end{aligned}\right\} \quad . \quad . \quad . \quad (7)$$

The scale-value constant a should be sensibly identical for all the exposures of any one night. If the telescope is really fixed with respect to the Earth, b, p, and q will also be constant; but if it is unstable, they may vary. The influence of diurnal aberration and of the variation of latitude (so far neglected) may be included at this point, for the former is equivalent to a shift of the whole field eastward by 0″·31 cos ϕ, and the latter to a small shift of the telescope relative to the Earth's axis, negligible during any one night, and, for longer intervals, likely to be inextricably confused

* The second-order terms in this expression are correct if it is assumed that the plate is "squared on" to the line of sight towards the *refracted* position of the pole (which is likely to be the case). If the plate is "squared on" to the unrefracted pole, Turner's formulæ give for the second-order terms in x, $\beta\cot\phi\{y^2 + x^2(1+\operatorname{cosec}^2\phi)\}$, and in y, $+\beta xy\cot\phi$. The differences between the two formulæ are practically insensible under the assumed circumstances.

with the motions of the pier, and even of the ground, on which the instrument is mounted.

§ 6. *Fundamental Equations.*—It is easy to choose the axes of measurement so that p, q are small, and a, b very small, so that their products by P, Q, p, q, r_0 are insensible. We may then combine equations (4), (6), and (7), and write

$$\left.\begin{aligned} x-\Delta-X' &= (a-\beta-\epsilon)X' + bY' - P\cos(\theta-\alpha_0) \\ &\quad - Q\sin(\theta-\alpha_0) + p + r_0 + \delta r + s \\ y-Y' &= (a-\beta-\epsilon)Y' - bX' - Q\cos(\theta-\alpha_0) \\ &\quad + P\sin(\theta-\alpha_0) + q \end{aligned}\right\} \quad (8)$$

where

$$\left.\begin{aligned} X' &= X_0\cos(\theta-\alpha_0) + Y_0\sin(\theta-\alpha_0) \\ Y' &= Y_0\cos(\theta-\alpha_0) - X_0\sin(\theta-\alpha_0) \end{aligned}\right\} \quad (9)$$

The refraction correction Δ which appears in these equations is, strictly speaking, a function of x', y'; but, as it is small, it may be taken from the table with x, y as arguments, without sensible error. The quantities s are in all ordinary cases very small (less than $0''.1$). If we neglect them for the present, the equations (8) represent an ordinary "four-constant" transformation between the co-ordinates X′, Y′ and $x-\Delta$, y.

Writing them in the form

$$x-\Delta-X' = AX' + BY' + C; \quad y-Y' = AY' - BX' + F \quad (10)$$

we may find, for each exposure, the constants A, B, C, F and their probable errors, by familiar methods. The values of C and F then give equations which, with the aid of (2), may be put in the form

$$\left.\begin{aligned} P'\cos(\theta-\alpha_0) + Q'\sin(\theta+\alpha_0) - (p+r_0) \\ \quad = C + \delta r - \delta P\cos(\theta-\alpha_0) - \delta Q\sin(\theta-\alpha_0) \\ Q'\cos(\theta-\alpha_0) - P'\sin(\theta-\alpha_0) - q \\ \quad = F - \delta Q\cos(\theta-\alpha_0) + \delta P\sin(\theta-\alpha_0) \end{aligned}\right\} \quad (11)$$

in which the second members are known, the terms in δP and δQ being almost insensible.

By turning the axes through an angle $\alpha_0-\theta$, these equations may be put in the alternative forms

$$\left.\begin{aligned} (x-\Delta)\cos(\theta-\alpha_0) - y\sin(\theta-\alpha_0) - X_0 &= A'X_0 + B'Y_0 + C' \\ (x-\Delta)\sin(\theta-\alpha_0) + y\cos(\theta-\alpha_0) - Y_0 &= A'Y_0 - B'X_0 + F' \end{aligned}\right\} \quad (10a)$$

$$\left.\begin{aligned} P' - (p+r_0)\cos(\theta-\alpha_0) + q\sin(\theta-\alpha_0) \\ \quad = C'\cos(\theta-\alpha_0) - F'\sin(\theta-\alpha_0) + \delta r\cos(\theta-\alpha_0) - \delta P \\ Q' - (p+r_0)\sin(\theta-\alpha_0) - q\cos(\theta-\alpha_0) \\ \quad = C'\sin(\theta-\alpha_0) + F'\cos(\theta-\alpha_0) + \delta r\sin(\theta-\alpha_0) - \delta Q \end{aligned}\right\} \quad (11a)$$

In the equations (11) or (11*a*), P′ and Q′ are constants; but p and q depend on the stability of the instruments, and may vary with the temperature, the time, and the like. Terms expressing such variations may be introduced into the equations as seems desirable, and the system may then be solved, and the position of the pole found.

Many other ways of solving the problem are possible; *e.g.* the equations (8) might be solved separately for each star, using all the exposures, with the unknowns $X_0 - P$, $Y_0 - Q$, $p + r_0 + s$, and q. But, in the nature of the case, the co-ordinates of the stars, relative to one another, can be determined much more accurately than the co-ordinates relative to the pole, since the latter are liable to errors due to anomalous refraction, motion of the telescope, etc., while the former are unaffected by these. It seems desirable, therefore, to introduce the former co-ordinates into the discussion as early as possible, so that we may be able to distinguish between the ordinary errors of photographic measurements—which affect the relative positions of the stars—and the errors peculiar to the present problem. The method of treatment here proposed saves considerable labour, for only one solution of the equations involving the times of observation is made for each plate, instead of one for each star.

§ 7. *Solution for each Exposure.* — It is necessary, to be sure, to solve a set of equations of the form (10) for each exposure, but this can be done very simply. The theory of such equations has often been discussed—for example, by Plummer * and Schlesinger.† The constants C and D are most accurately determined when the "centre of gravity" of the reference stars falls at the origin—in this case, at the pole. It should usually be easy to choose these stars so that this condition shall be very nearly fulfilled, and it may be exactly satisfied by giving reduced weight to two or three of the stars farthest from the pole; a process which, as Schlesinger points out, is really less arbitrary than deciding not to measure them at all. When the (weighted) centre of gravity of the reference stars is at the pole, the constants C and F are simply the weighted means of the numerical terms of the equations (10), and a formal solution is superfluous.

If an isolated plate is to be discussed, it is convenient to choose the measured co-ordinates for any one exposure (corrected for refractional distortion) as the standard to which to reduce the others, using the equations (10*a*) and setting a_0 equal to the sidereal time of the chosen exposure. In this case the constants A′ and B′ should be quite insensible (provided that the times of exposure are accurately known), and the first members of (10*a*) should be the same for all the stars for a given exposure. Any systematic "run" with the co-ordinates is evidence that the plate has shifted its position during the night, and is worthless. The weighted means of these quantities are the numbers C′ and F′ which appear in (11*a*). For the "standard" exposure we have $C' = F' = 0$, with the same weight as the other values. The differences between C′ and F′ and the quantities for the individual stars for each exposure arise from two sources: the errors of the individual measured co-ordinates x, y, and those of the assumed standard co-ordinates X_0, Y_0. By adding to the assumed co-

* *Monthly Notices*, vol. lxiv. pp. 645–648, 1904.

† *Astrophysical Journal*, vol. xxxiii. pp. 161–172, 1911.

ordinates of each star the mean of all the residuals for this star for all the exposures, including the zero residual for the standard exposure, we will evidently obtain an improved set of co-ordinates of the stars, representing the mean of all the measures of the plate. These, together with the values of P′ and Q′ derived from (11*a*), constitute the final results from the plate.

§ 8. *Choice of Standard.*—If several plates are to be reduced, it will be of advantage to make one of the exposures on each at the same sidereal time—such as 0^h, 6^h, 12^h, or 18^h—and choose these as the standard exposures. The final co-ordinates derived from all the plates will then have very nearly the same scale-value and orientation, and the mutual comparison of the plates can be made by simple "four-constant" solutions.

If a long series of plates are to be discussed, it will probably save labour, after four or five plates have been reduced as above, to adopt the system of relative co-ordinates of the stars obtained from the mean of these as a permanent standard for use in the reductions, and use the equations (10) and (11) in reducing the rest of the plates. The principal saving would be in the formation of the quantities $X' = X_0 \cos(\theta - \alpha_0) + Y_0 \sin(\theta - \alpha_0)$, and the like, which, since X_0 and Y_0 are now constants, can be tabulated with $(\theta - \alpha_0)$ as argument. By a suitable choice of α_0, any error of orientation of the plate in the measuring apparatus can be corrected, so that the differences $x - \Delta - X'$ are sensibly constant for each exposure, as before. When a long series of plates has been reduced, the residuals for the reference stars may be grouped according to the sidereal time of exposure, and the means discussed. If there is no optical distortion of the field, the mean values should be representable by the expression

$$\begin{aligned} dX_0 \cos(\theta - \alpha_0) + dY_0 \sin(\theta - \alpha_0) + s &= \text{mean residual in } x \\ dY_0 \cos(\theta - \alpha_0) - dX_0 \sin(\theta - \alpha_0) &= \text{mean residual in } y \end{aligned}$$

where dX_0, dY_0 are final corrections to the previously adopted standard co-ordinates, and s the correction to the refraction arising from the colour of the star. If optical distortion exists, it will be revealed by a study of the residuals after this adjustment. It is obvious that a radial distortion will only produce constant errors in the deduced polar distances of the stars, which cannot be detected until comparison is made with plates having a different centre. Other forms of distortion may in general be expressed by a series of the form

$$\Delta x = ax + by + c(x^2 + y^2) + d(x^2 - y^2) + exy + \quad . \quad (12)$$

The effect of the distortion upon the constants C and F is the mean of the values of Δx (or Δy) for the reference stars. If the centre of gravity of these stars is at the pole, the first-order terms will vanish in this mean, and, if the stars are symmetrically distributed, the terms involving $(x^2 - y^2)$ and xy will be very small. The term involving $(x^2 + y^2)$ is constant for each star, wherever its

image falls on the plate, and merges with the atmospheric dispersion term s. Its effect on C is constant, and it will not influence the deduced co-ordinates of the pole at all. The terms of higher order in the expansion (12) will be similar in type to those already discussed. It therefore appears that optical distortion, and differences of refraction for the different stars, will be without sensible effect on the computed position of the pole, provided that the reference stars are properly chosen.

§ 9. *Solution for the Position of the Pole.*—With the solution of the equations (11) or (11*a*) we come to the crux of the whole problem. If the telescope and the pier upon which it is mounted remain fixed, in all their parts, relative to the Earth as a whole, throughout the night, $p+r_0$ and q will be constants, and the solution is very simple. It is easy to show that in this case P′, Q′, p, and q are all determined with the weight $n(1-d^2)$, where n is the number of exposures, and d the ratio of the distance of the centre of gravity of all the images of any star from the pole to the polar distance of the star. Unless the exposures cover an interval of at least eight hours, the resulting weights are small. It will therefore be possible to determine the position of the pole much more accurately in winter than in summer.

If, however, we admit the possibility that the plate-holder, or the telescope, or the pier, may have moved during the night, the solution becomes difficult; for these motions may or may not be linear, may depend on the time, the temperature, etc.

Whether or not the instrument is really stable can be determined only by reducing a number of plates, on the assumption that it did not move. If the values then obtained for the probable error of one of the equations (11) agree, on the average, within reasonable limits, with the probable errors found from the solutions of (10) for the quantities C and F, which form the essential parts of the absolute terms of (11); and if, in addition, the positions of the pole found on different nights, after correction for aberration, precession, and nutation, agree with one another within the limits assigned by their probable errors, we may then be satisfied of the stability of our apparatus. In applying the first of these tests, it must be borne in mind that there are certain real causes, other than motion of the instrument, which may introduce small errors into C and F—for example, anomalous changes in refraction during the night,—and that therefore a moderate excess of the probable errors of these quantities, determined from (11) above the values found from (10), is no cause for alarm.

If the instrument is not stable, the chance of getting reliable results is very small. Even if the law of its motion was known, the introduction into (11) of additional terms (proportional, for example, to the time) diminishes enormously the weight of the deduced co-ordinates of the pole,—though it might help matters somewhat if exposures could be made on the plate on two successive days, taking all precautions not to disturb it in the interval. If the law of the motion is not known, the situation is hopeless.

Any arbitrary displacement of the plate centre may be expressed, for the day of observation, by Fourier series of the form

$$p = p_0 + a_1 \cos(\theta - \alpha_0) + a_2 \cos 2(\theta - \alpha_0) + \ \ldots \\ + b_1 \sin(\theta - \alpha_0) + b_2 \sin 2(\theta - \alpha_0) \ \ldots$$

$$q = q_0 + c_1 \cos(\theta - \alpha_0) + c_2 \cos 2(\theta - \alpha_0) + \ \ldots \\ + d_1 \sin(\theta - \alpha_0) + d_2 \sin 2(\theta - \alpha_0) + \ \ldots$$

Introducing these expressions into (11*a*) it appears at once that, even if all the periodic terms were determined, there would remain additions to P and Q, of magnitude $\frac{1}{2}(a_1 - d_1)$ and $\frac{1}{2}(b_1 + c_1)$, which remain essentially inextricable from these quantities, and whose magnitude cannot be found from the other coefficients of the series.

In any such case, it would probably be much more worth while to devote one's efforts to making the telescope more nearly immovable, rather than to attempting to deduce results from plates vitiated by its motion.

Princeton University Observatory:
1913 *February* 21.

Observations of the Later Spectrum of Nova Geminorum No. 2.

By W. S. Adams, M.A., and J. B. Lasby.

In view of Professor Fowler's recent laboratory investigations of the first line of the principal series of hydrogen at λ 4686, the following observations of the variation in intensity of this line in the spectrum of Nova Geminorum are of interest.

A photograph taken on May 5, 1912, showed a bright maximum at λ 4695 involved in a long expanse of continuous spectrum. On May 10 another maximum had appeared at λ 4679, and the two with the intervening structure formed a fairly well-defined band with its centre at λ 4686. At this time the band was slightly more intense than the nebular band at λ 4959. Apparently λ 4686 then became considerably fainter, as on photographs taken in August and September there are merely traces of the two maxima. On November 16, however, a photograph showed the band to be strong and well defined, having fully one-half the intensity of the Hβ band. Our last negative, taken on April 27, 1913, shows the band to be of almost exactly the same intensity as Hβ.

The great increase in intensity of λ 4686 shown by the November 16 photograph is of especial interest when considered in connection with Professor Barnard's observation that the brightness of the star increased 0·7 magnitude betwen November 15 and 17. This makes it fairly probable that the change in intensity of the line was a very sudden one.

The structure of the λ 4686 band on all of our recent photographs is precisely like that of the hydrogen and the nebular bands. It consists of two equal maxima at the edges of the band

separated by a broad faint absorption band which lies symmetrically upon it. The presence of absorption at such a late stage in the development of the Nova spectrum is a most interesting feature.

The widths and the positions of the centres of both the hydrogen and the nebular bands have remained remarkably constant throughout our entire series of observations, allowance being made for the variation in the density of the negatives, for the increase in intensity of the nebular bands, and for the probable slight decrease in intensity of the hydrogen bands. The following brief table illustrates this for three of the most important lines. Δ is the displacement of the centre of the band.

	Hβ.		λ 4959.		λ 5007.	
Date of Photograph.	Width.	Δ.	Width.	Δ.	Width.	Δ.
1912, May 10	30	+1·6	26	+1·1	30	+0·6
Aug. 18	26	+1·0	25	+1·3	28	+1·1
Sept. 4	25	+1·2	26	+0·8	29	+0·8
Nov. 16	27	+1·4	27	+1·2	29	+1·3
1913, April 27	24	+1·3	25	+0·8	29	+0·9

The photograph of November 16, 1912, still showed the presence of a considerable amount of continuous spectrum, but that of April 27, 1913, showed a mere trace, the spectrum consisting almost wholly of detached bright bands. From this point of view it may be said to have reached the true nebular stage. A list of the bands observed may be of interest.

Band.	Limits.	Width.	Centre.	Remarks.
Hδ	4092–4113	21	4102	
Hγ	4332–4354	22	4343	These two bands are not separated.
Nebular	4354–4376	22	4365	
Nebular	4505–4535	20	4518	Very ill-defined.
Nebular	4593–4617	24	4605	
Nebular	4627–4654	30	4640	
H′	4675–4699	24	4687	
Hβ	4851–4875	24	4863	
Nebular	4947–4972	25	4960	
Nebular	4993–5022	29	5008	

In view of the identification of several bright bands, in particular those near λ 4925 and λ 5019, in the earlier spectrum of the Nova with enhanced lines of iron by Butler, Baxandall, and Newall and Stratton, perhaps a few words may be said about the evidence for the presence of helium in the star. On a photograph taken on May 10, 1912, bright bands were measured with centres at

$$\lambda\,4473,\ \lambda\,5049,\ \lambda\,5878,\ \text{and}\ \lambda\,6677\pm,$$

to which, so far as we know, no origin other than helium can be

ascribed with any reasonable probability, there being no enhanced lines in these positions. On this photograph the two bands in question were visible, the centre of the first being determined as λ 4924·3, while the second could not be measured on account of the presence of the very strong nebular band at λ 5008. No trace could be seen on the negative of any band near λ 5316, where there is a very strongly enhanced iron line and one of the most prominent chromospheric lines in the entire solar spectrum. It seems to us, therefore, that the evidence for the epoch of May 10 is distinctly in favour of the view that at that time the bands in question were due to helium.

To compare with these results we have a photograph taken on March 28, 1912, or fifteen days after the discovery of the Nova by Enebo. On this negative the presence of helium is attested by two bright bands at λ 4473 and λ 5878, although the weaker bands at λ 5049 and λ 6677 could not be measured. Moreover, a faint bright band at λ 5320 was observed, which, as Mr. Baxandall has suggested, is probably due to the enhanced iron line at λ 5316. It seems to us probable, therefore, that at this early date the bands at λ 4926 and λ 5021 were due to a blend of helium and enhanced iron, since they are relatively too strong to be due to either element alone. Shortly afterwards the iron begins to disappear, and by May 10 the bands are due to helium alone. This view is strengthened by the change in wave-length of the λ 4926 band. On March 28, measures gave the position of its centre as λ 4926·2, while on May 10, as already stated, the position was λ 4924·3. This difference corresponds in direction to that suggested, the wave-length of the helium line being considerably shorter than that of the enhanced iron line.

Mount Wilson Solar Observatory:
1913 *June.*

Erratum.

Page 605, line 3 of *Correction, for* vol. liv. *read* vol. lvi.

LISTS OF ADDITIONS

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